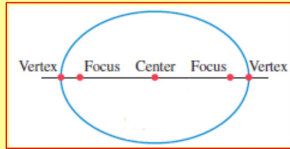


8.2 Ellipses

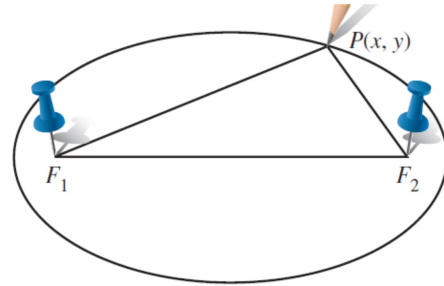


ESSENTIAL QUESTIONS

1. How do you find the equation, the vertices, the foci, and the eccentricity of an ellipse?
2. Where in science and nature do you see or use ellipses?

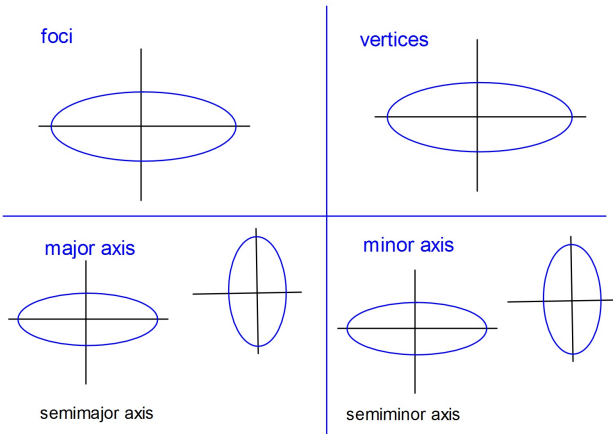
Definition of ellipse

An ellipse is the set of all points whose distances from 2 fixed points (foci) have a constant sum.



http://www.sciences.univ-nantes.fr/sites/genevieve_tulloue/conics/drawing/ellipse_string.html

Terminology:



Ellipses that are elongated horizontally will have standard form

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

where (h, k) is the center and each vertex is a units left or right of the center, and $a > b$.

Ellipses that are elongated vertically will have standard form

$$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$$

where (h, k) is the center and each vertex is a units up or down from the center, and $a > b$.

The **foci** (plural of focus) are located c units from the center along the **major axis**.

This Pythagorean relationship will help you find the value of c :

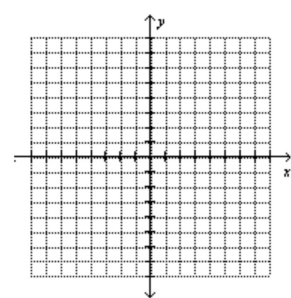
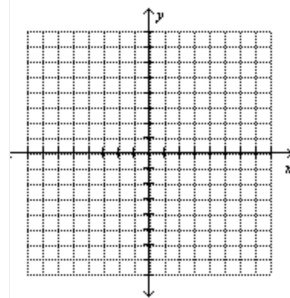
$$a^2 = b^2 + c^2$$

Note that a is by itself because it is greater than either b or c !

Graph. Then find the foci and vertices.

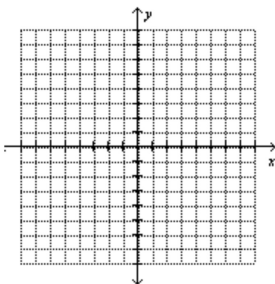
$$\frac{(x-1)^2}{25} + \frac{(y+2)^2}{16} = 1$$

$$\frac{x^2}{36} + \frac{(y-3)^2}{49} = 1$$

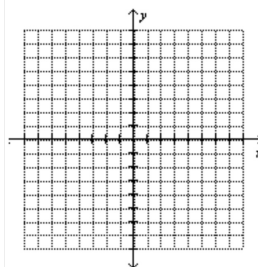


Put the equation in standard form. Then find its center, vertices, & foci. (Graphing will help!)

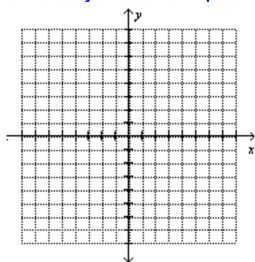
$$3x^2 + 5y^2 - 12x + 30y + 42 = 0$$



Write an equation for the ellipse with foci $(0, \pm 3)$ and minor axis length 6.



Write an equation for the ellipse with foci $(-2, 1)$ and $(-2, 5)$ and major axis endpoints $(-2, -1)$ and $(-2, 7)$.



The **eccentricity**, e , of an ellipse refers to how fat or thin the ellipse is.

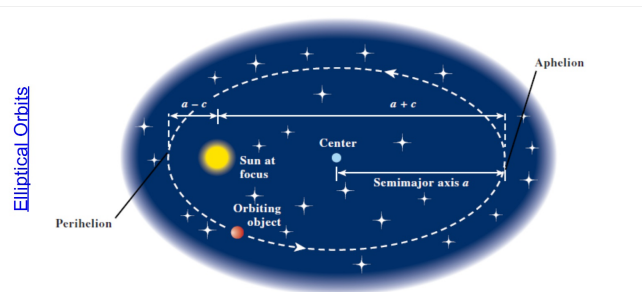
$$e = \frac{c}{a}$$

As $e \rightarrow 1$, the ellipse becomes narrower.
As $e \rightarrow 0$, the ellipse becomes fatter (almost circular)

Note: The e that we use to represent eccentricity is NOT the same e that is the base of the natural log function.

Prove the graph of the equation is an ellipse. Then find its eccentricity.

$$4x^2 + y^2 - 32x + 16y + 124 = 0$$



The Moon's Orbit The Moon's apogee (farthest distance from Earth) is 252,710 miles, and perigee (closest distance to Earth) is 221,463 miles. Assuming the Moon's orbit of Earth is elliptical with Earth at one focus, calculate and interpret a , b , c , and e .