7.7 Use L'Hôpital's Rule to evaluate limits that have indeterminate form.

ESSENTIAL QUESTION: How do we know when a limit is in indeterminate form?

Consider these limits. Why does direct substitution fail?

$$\lim_{x\to 1}\frac{x^2-1}{x-1}$$

$$\lim_{x\to\infty}\frac{5x^2+x-1}{2x^2-x+1}$$

What techniques would you use to evaluate the limits?

L'Hôpital looked at two functions f and g that are continuous on [a, b] and differentiable on (a, b), except possibly at a point c in [a, b].

$$\lim_{x\to c}\frac{f(x)}{g(x)}$$

He found that sometimes the limit of their quotients produces an indeterminate form such as

$$\frac{0}{0}, \frac{\infty}{\infty}, \frac{-\infty}{\infty}, \text{ or } \frac{\infty}{-\infty}$$

An indeterminate form doesn't necessarily mean that the limit doesn't exist.

What L'Hôpital found is known as

L'Hôpital's Rule

If the limit of the quotient of f and g produces an indeterminate form, then

$$\lim_{x\to c}\frac{f(x)}{g(x)}=\lim_{x\to c}\frac{f'(x)}{g'(x)}$$ $g'(x)\neq 0$

NOTE: This is NOT the quotient rule.

1. $\lim_{x\to -1}\frac{2x^2-x-3}{x+1}=$

2. $\lim_{x\to 0}\frac{\sin 4x}{2x}=$

L'Hopital's Rule can be repeated as long as each step produces an indeterminate form.

3. $\lim_{x\to\infty} \frac{x^2}{e^x} =$

Other indeterminate forms such as

$$0^0,\ 0^\infty,\ 0\cdot\infty,\ \infty\cdot\infty,\ 1^\infty,\ \infty-\infty......$$

must be rearranged to get into this form:

$$\frac{0}{0}, \frac{\infty}{\infty}, \frac{-\infty}{\infty}, \text{ or } \frac{\infty}{-\infty}$$

4. $\lim_{n\to 2^+} \left(\frac{1}{x^2-4} - \frac{\sqrt{x-1}}{x^2-4} \right) =$

5. $\lim_{n\to\infty} (1+x)^{1/x} =$

6. $\lim_{n\to\infty} x \tan \frac{1}{x} =$