7.4
Decomposing Rational Expressions Using Partial Fractions

ESSENTIAL QUESTION

How is a rational expression decomposed into the sum of 2 or more rational expressions that are simpler than the original?

Combine these rational expressions into a single fraction:

$$\frac{2}{x}+\frac{1}{x-1}-\frac{4}{x+1}$$

Sometimes it's necessary to "decompose" a rational fraction into simpler expressions so that we can integrate expressions in calculus. Decomposition "undoes" the process you just did in the example above.

DECOMPOSITION

Decompose $\frac{1}{x^2+3x-18}$

Begin by factoring the denominator:

Let $\frac{1}{(x+6)(x-3)}=\frac{A}{x+6}+\frac{B}{x-3}$

Multiply by the common denominator:

Choose appropriate values of x to find A and B:

Now rewrite the original fraction in its decomposed form:

Decompose: $\frac{4x-11}{2x^2-x-3}$

Sometimes the denominator factors so that there is a **repeated linear factor.**

$$\frac{4x+4}{x^3+2x^2}$$

Try this one:

$$\frac{-6x+25}{x^3-6x^2+9x}$$

Decomposing with an irreducible quadratic factor:

$$\frac{x^2+4x+1}{x^3-x^2+x-1}$$

Here's another. Remember that $a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$.

$$\frac{2x^2-4x+3}{x^3+1}$$