7.3 Trig Integrals with Powers of Secant and Tangent

$$\int \sec^m(ax)\cdot\tan^n(ax)dx$$

ESSENTIAL QUESTION: What trig identities may be helpful when integrands contain powers of secant and tangent?

CASE 1: Even power of secant

Save $\sec^2 x$ and convert the rest to tanx using the identity $\tan^2 x + 1 = \sec^2 x$.

1. $\int \sec^4 x\sqrt{\tan x}\,dx$

CASE 2: Odd powers of tangent

Save secx·tanx and convert the rest to secx using the identity $\tan^2 x + 1 = \sec^2 x$.

2. $\int \tan^3\left(\frac{\pi x}{2}\right)\cdot\sec^3\left(\frac{\pi x}{2}\right)dx$

CASE 3: Even power of tangent, no secant

Convert one $\tan^2 x$ to $(\sec^2 x - 1)$. Repeat if necessary.

3. $\int \tan^6 x\,dx$

CASE 4: Odd power of secant, no tangent.

Use integration by parts with dv = $\sec^2 x$.

4. $\int \sec^3 x\,dx$

And finally.....

CASE 5: When none of these work, convert everything to sines and cosines!