7.2 Integration by Parts

ESSENTIAL QUESTION:
Under what circumstances is integration by parts an inappropriate method for integrating?

We use integration by parts (IBP) for integrands involving products of unrelated algebraic and transcendental functions.

$$\int x \ln x \, dx \qquad \int x^2 e^x \, dx \qquad \int e^x \sin x \, dx$$

IBP is based on the product rule for derivatives.
Let u and v be functions of x.

$$\frac{d}{dx}[uv] = u\frac{dv}{dx} + v\frac{du}{dx}$$

If u' and v' are continuous, we can integrate both sides:

$$\int \frac{d}{dx}[uv] = \int u\frac{dv}{dx}dx + \int v\frac{du}{dx}dx$$

$$uv = \int u\,dv + \int v\,du$$

With a little rearranging, we have the rule for integration by parts:

$$\boxed{\int u\,dv = uv - \int v\,du}$$

Here is an acronym that may prove helpful when trying to decide which function in the integrand should be **u.** **LIATE**

L = logarithm
I = inverse trig
A = algebraic
T = trig
E = exponential

Whichever function comes first in the acronym should be defined as **u.**

1. $\int xe^{2x}\,dx$

2. $\int \ln x \, dx$

3. $\int x^3 \ln x \, dx$

4. $\int \theta \sec\theta \tan\theta \, d\theta$

Why is IBP not appropriate for this integral?

$$\int x\sqrt{x-3}\, dx$$

This one requires repeated applications of IBP.

5. $\int x^2 e^{-x} dx$

You can do the same problem using the tabular method.

$$\int x^2 e^{-x} dx$$

Remember, we let u = x^3 and dv = e^{-x}.

Alternating sign	u & its derivatives	dv and its integrals
+	x^2	e^{-x}

Only use the tabular method when one of the factors is a power of x!

This one requires repeated applications of IBP...
with a twist!

6. $\int e^x \sin 2x \, dx$