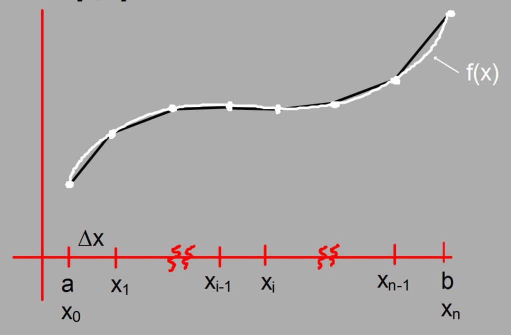
6.4 Length of a Curve (Arc Length)

ESSENTIAL QUESTION:
What are some real-life instances for which we might need to find the length of a curve?

Suppose we want to find the length of the curve f(x) on the interval [a, b].



Imagine using the distance formula repeatedly on very small adjacent intervals.

Let s = arc length.

$$s \approx \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2} + \cdots + \sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2} + \cdots + \sqrt{(x_n - x_{n-1})^2 + (y_n - y_{n-1})^2}$$

$$= \sum_{i=1}^{n} \sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2}$$

$$= \sum_{i=1}^{n} \sqrt{(\Delta x_i)^2 + (\Delta y_i)^2} = \sum \frac{\sqrt{(\Delta x_i)^2 + (\Delta y_i)^2}}{\sqrt{(\Delta x_i)^2}} \cdot \sqrt{(\Delta x_i)^2}$$

$$= \sum_{i=1}^{n} \sqrt{1 + \left(\frac{\Delta y_i}{\Delta x_i}\right)^2} \cdot \Delta x_i$$

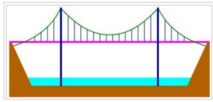
Finally, $s = \lim_{n \to \infty} \sum_{i=1}^{n} \sqrt{1 + [f'(x)]^2} \cdot \Delta x_i = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$

1. Find the length of $y = \dfrac{x^5}{10} + \dfrac{1}{6x^3}$ on the interval [1, 2].

***No calculator! This requires some neat algebra!

WHEW!!! Lucky for us, most arc length problems are calculator active. Set up the integral, then use your calculator to evaluate.

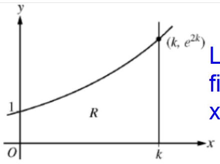
2. Find the length of $x = \sqrt{36 - y^2}$, $0 \le y \le 3$



A cable is hung between 2 bridge towers that are 200 feet apart. The cable takes the shape of a catenary whose equation is $y = 75(e^{x/150} + e^{-x/150})$

Find the length of the cable between the two towers.

For more on catenary curves:
http://mathforum.org/mathimages/index.php/Catenary



Let $f(x) = e^{2x}$. Let R be the region in the first quadrant bounded by the graph of f, x = 0, y = 0, and x = k, where $k > 0$.

a. Write, but do not evaluate, an expression involving an integral that gives the perimeter of R in terms of k.

b. The region R is rotated about the x-axis to form a solid. Find the volume V of the solid, in terms of k.

c. The volume V, found in part (b), changes as k changes. If $\frac{dk}{dt} = \frac{1}{3}$, determine $\frac{dV}{dt}$ when $k = \frac{1}{2}$.