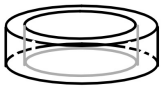
6.2 Volume by Washer Method

ESSENTIAL QUESTION:
How can you tell when you need to use the disc method or the washer method for volume?

In order to come up with an integral for volume, we must first determine how to find the volume of a washer.



Volume of cylinder = $\pi r^2 h$

Volume of washer = vol. of outer cyl. - vol. of inner cyl.

$= \pi R^2 h - \pi r^2 h$

$= \pi(R^2 - r^2)h$

Note that **h = dx or dy**!

Let's set up a definite integral to calculate the volume.

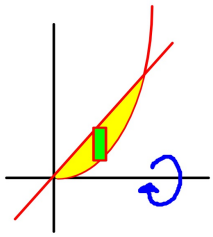
Let R = the distance from the axis of revolution to the outer curve.
Let r = the distance from the axis of revolution to the inner curve.

$$V = \pi \int_a^b \left(R^2 - r^2\right) dx$$ (Use when representative rectangle has width dx.)

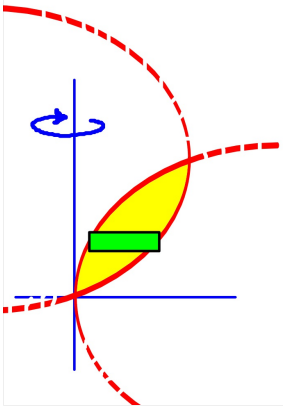
$$V = \pi \int_c^d \left(R^2 - r^2\right) dy$$ (Use when representative rectangle has width dy.)

Examples:

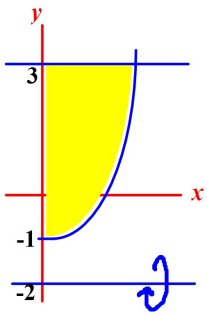
1. Revolve the region enclosed by the graphs of $y = x^2$ and $y = x$ about the x-axis. Find the volume of this solid.



2. Find the volume of the region bounded by the graphs of $x = 4y - y^2$ and $x = y^2$ about the y-axis.

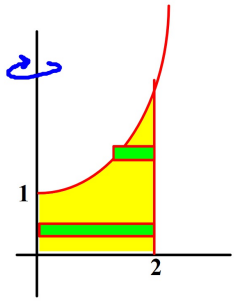


3. Revolve the region enclosed by $y = x^2 - 1$, $x = 0$, and $y = 3$ about the line $y = -2$. Find the volume of this solid.



This one presents a unique problem....

4. Find the volume of the solid generated when the regions enclosed by the graphs of $y = x^2 + 1$ and $x = 2$ are rotated about the y-axis.



Try this:

Let R be the region in the first and second quadrants bounded above by the graph of $y = \frac{20}{1+x^2}$ and below by the horizontal line $y = 2$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the x-axis.