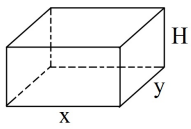
# 6.2
# Volume by Cross Sections

ESSENTIAL QUESTION:
How does integrating the area function for a particular cross section result in the volume of a solid with known cross sections?

Recall from geometry that we can find the volume of a prism by multiplying the area of the base of the prism by its height.



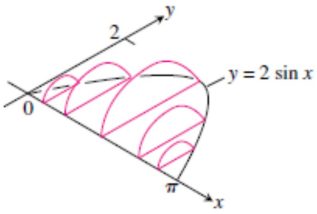

$V = B \cdot H$

This formula works for solids with bases of any shape, and it can be applied to solids with cross sections of similar shapes.

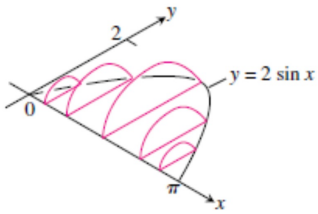
We can use this idea to find the volume of a solid when we know the shape of its base and the shape of its cross sections.

Exploration:

A mathematician has a paperweight made so that its base is the shape of the region between the $x$-axis and one arch of the curve $y = 2 \sin x$ (linear units in inches). Each cross section cut perpendicular to the $x$-axis (and hence to the $xy$-plane) is a semicircle whose diameter runs from the $x$-axis to the curve. (Think of the cross section as a semi-circular fin sticking up out of the plane.) Find the volume of the paperweight.




How can we approach this? Think Riemann Sums!




## Volume by cross sections

When the cross sections are perpendicular to the x-axis, the area function is A(x).

$$V = \int_a^b A(x)\,dx$$

When the cross sections are perpendicular to the y-axis, the area function is A(y).

$$V = \int_c^d A(y)\,dy$$

## To determine the area function:

1. Describe the length of the edge of the cross section that lies in the base in terms of x (or y).
2. Chose an appropriate area formula from geometry for the shape of the cross section.
3. Write an area function A(x) or A(y).
4. Integrate the area function on the appropriate interval.

Some area formulas you will need to know:

Square: $A = s^2$

Rectangle: $A = bh$

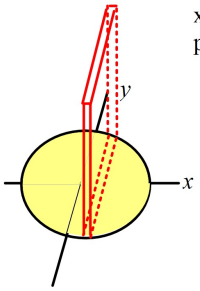
Triangle: $A = \frac{1}{2}bh$

Equilateral triangle: $A = \frac{s^2\sqrt{3}}{4}$

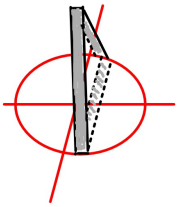
Circle: $A = \pi \cdot r^2$

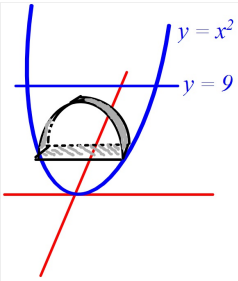
Semicircle: $A = \frac{1}{2}\pi r^2$

1. Find the volume of a solid with base $x^2 + y^2 = 4$ and square cross sections perpendicular to the x-axis.




2. Find the volume of a solid with base $x^2 + y^2 = 4$ with cross sections that are equilateral triangles perpendicular to the x-axis.






3. The base of a solid is bounded by the graphs of $y = x^2$ and $y = 9$. Cross sections are semicircles. Find the volume of the solid.