5.5 Bases Other Than e

ESSENTIAL QUESTIONS:

How are the derivatives and integrals of log and exponential functions (with bases other than e) derived from those with base e?

Change of Base

$$\log_a x = \frac{\log x}{\log a} = \frac{\ln x}{\ln a}$$

Inverse functions

If $f(x) = a^x$, then $f^{-1}(x) = \log_a x$

Develop a rule for finding the derivative of $y = a^x$:

Develop a rule for finding the derivative of $y = \log_a x$:

Derivative of functions with bases other than e

1. $f(x) = a^x \Rightarrow f'(x) = (\ln a)a^x$
2. $f(x) = a^u \Rightarrow f'(x) = (\ln a)a^u \cdot u'$
3. $f(x) = \log_a x \Rightarrow f'(x) = \frac{1}{(\ln a)x}$
4. $f(x) = \log_a u \Rightarrow f'(x) = \frac{u'}{(\ln a)u}$

Examples: Find the derivatives.

1. $g(x) = 2^{-x}$

2. $y = x(7^{-3x})$

Integrals

$$\int a^x \, dx = \frac{1}{\ln a} a^x + C$$

4. $\int 2^{3x-4} \, dx$

5. $\int 2x^2\left(5^{x^3}\right)dx$

6. In a group project about learning theory, a mathematical model for the proportion P of correct responses after n trials was found to be

$$P = \frac{0.86}{1+e^{-0.25n}}$$

a) Find $\lim_{n\to\infty} P$

b) Find the rate at which P is changing when $n = 3$.