5.4 Multiple Angle Identities

ESSENTIAL QUESTION:
How are the double-angle, power reducing, &half-angle identities used to solve problems and verify identities?

DOUBLE-ANGLE IDENTITIES

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\begin{aligned}\cos 2\theta &= \cos^2\theta - \sin^2\theta \\ &= 2\cos^2\theta - 1 \\ &= 1 - 2\sin^2\theta\end{aligned}$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

We can use angle sum/difference identities to prove the double angle identities:

Prove $\sin 2\theta = 2\sin\theta\cos\theta$

Prove $\tan 2u = \dfrac{2\tan u}{1 - \tan^2 u}$

examples:

Rewrite in terms of sin θ and cos θ: sin(3θ) + cos(2θ)

Solve on the interval $[0, 2\pi)$.

$\sin 2x = \sin x$

Solve on the interval $[0, 2\pi)$.

$2\cos^2 x + \cos x = \cos 2x$

POWER REDCTION IDENTITIES

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\tan^2\theta = \frac{1-\cos 2\theta}{1+\cos 2\theta}$$

examples:

Prove $\cos^3 x = \left(\frac{1}{2}\cos x\right)(1 + \cos 2x)$

HALF-ANGLE IDENTITIES

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$$

examples:

Find the exact value using half-angle identities.

$\cos(\pi/8)$ $\sin(5\pi/12)$

Use a half-angle identity to solve for x on the interval $[0, 2\pi)$.

$\sin^2 x = \cos^2\left(\frac{x}{2}\right)$