

5.1 Fundamental Trig Identities

ESSENTIAL QUESTION:
How are basic trig
identities used to
simplify expressions
and solve trig equations?

Basic Trigonometric Identities

Reciprocal Identities

$$\csc \theta = \frac{1}{\sin \theta} \quad \sec \theta = \frac{1}{\cos \theta} \quad \cot \theta = \frac{1}{\tan \theta}$$

$$\sin \theta = \frac{1}{\csc \theta} \quad \cos \theta = \frac{1}{\sec \theta} \quad \tan \theta = \frac{1}{\cot \theta}$$

Quotient Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

You'll need to memorize these if you haven't already!

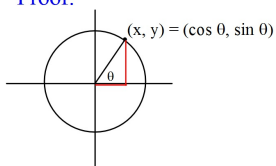
Pythagorean Identities

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$\cot^2 \theta + 1 = \csc^2 \theta$$

Proof:



Examples:

Find $\sin \theta$ and $\cos \theta$ if $\tan \theta = 5$ and $\cos \theta > 0$.

Find $\sin \theta$ and $\tan \theta$ if $\cos \theta = 0.8$ and $\tan \theta < 0$.

Cofunction Identities

$$\sin \left(\frac{\pi}{2} - \theta \right) = \cos \theta \quad \cos \left(\frac{\pi}{2} - \theta \right) = \sin \theta$$

$$\tan \left(\frac{\pi}{2} - \theta \right) = \cot \theta \quad \cot \left(\frac{\pi}{2} - \theta \right) = \tan \theta$$

$$\sec \left(\frac{\pi}{2} - \theta \right) = \csc \theta \quad \csc \left(\frac{\pi}{2} - \theta \right) = \sec \theta$$

Odd-Even Identities

$$\sin(-x) = -\sin x \quad \cos(-x) = \cos x \quad \tan(-x) = -\tan x$$

$$\csc(-x) = -\csc x \quad \sec(-x) = \sec x \quad \cot(-x) = -\cot x$$

I'll give you these--no need to memorize!

Examples:

If $\tan(\pi/2 - \theta) = -5.32$, find $\cot \theta$.

If $\cos \theta = 0.34$, find $\sin(\theta - \pi/2)$.

Use identities to simplify trig expressions

With factoring:

Simplify the expression $\sin^3 x + \sin x \cos^2 x$.

Multiply first, then simplify using identities:

$$\frac{(\sec x + 1)(\sec x - 1)}{\sin^2 x}$$

Simplify by combining fractions and using identities:

$$\frac{\cos x}{1 - \sin x} - \frac{\sin x}{\cos x}$$

Solve trig equations using identities:

Note: It's okay to multiply both sides of an equation by a trig function, but NEVER DIVIDE (You'll lose a solution!)

Solve $\frac{\cos^3 x}{\sin x} = \cot x$ on the interval $[0, 2\pi)$.

Find all solutions to the equation

$$2\sin^2 x + \cos x = 1.$$

Find all solutions to the equation

$$\tan \theta = \cos \theta.$$