5.1 Derivatives of Natural Logs

ESSENTIAL QUESTION

If u = f(x), how do you find the derivative of y = ln u?

Remember the properties of natural logs?

1. ln ab = ln a + ln b
2. $\ln \frac{a}{b} = \ln a - \ln b$
3. $\ln x^n = n \ln x$
4. ln 1 = 0
5. ln e = 1

Example: Expand $\ln\left(\frac{3x\sqrt[4]{x^2-1}}{x-1}\right)$

We define the natural log function

$$\ln x = \int_1^x \frac{1}{t}\,dt$$

Domain: $x > 0$
Range: all real numbers

Using the Second Fundamental Theorem of Calculus,

$$\frac{d}{dx}[\ln x] = \frac{d}{dx}\left[\int_1^x \frac{1}{t}\,dt\right] = \frac{1}{x}$$

So we can conclude if $y = \ln x$,

then $$\frac{dy}{dx} = \frac{1}{x}$$

We can also generalize. Suppose u is a function of x, then if $y = \ln u$,

$$\frac{dy}{dx} = \frac{u'}{u}$$

Examples: Find the derivative of each function.

1. $y = \ln 3x$

2. $f(x) = \ln x^2$

For complicated expressions, the derivative will be easier if the original function is expanded.

3. $f(x) = \ln\sqrt{x^2 + 1}$

4. $y = \ln\left(\frac{2x}{x+3}\right)$

5. $y = \ln(\ln x)$

6. $y = \frac{\ln x}{x}$

For extremely complicated equations, we can use <u>logarithmic differentiation.</u>

1. Take ln of both sides.
2. Differentiate both sides. You'll have to differentiate ln(y) implicitly.
3. Solve for dy/dx.
4. Substitute for y.

Find dy/dx for $y = \sqrt{\frac{x^2 - 1}{x^2 + 1}}$