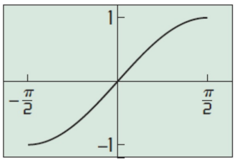
If $y = \sin\theta$, then $\sin^{-1} y = \theta$.

Functions and their inverses

1. A function has an inverse if and only if that function is one-to-one.
2. The graph of the inverse of f(x) is the reflection of f(x) in the line y = x.
3. The domain of f(x) becomes the range of $f^{-1}(x)$ and the range of f(x) becomes the domain of $f^{-1}(x)$.

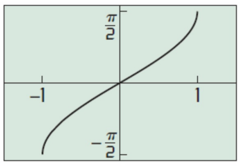
Since y = sin x is not one-to-one, we must determine an interval on which sin x takes on all of its range values. These are called the PRINCIPAL VALUES of sin x.



y = sin x

Domain: $[-\frac{\pi}{2}, \frac{\pi}{2}]$

Range: [-1, 1]

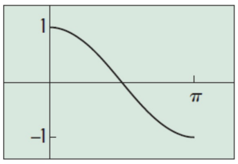


$y = \sin^{-1} x$ or y = arcsin x

Domain: [-1, 1]

Range: $[-\frac{\pi}{2}, \frac{\pi}{2}]$

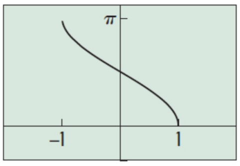
Since y = cos x is not one-to-one, we must determine an interval on which cos x takes on all of its range values. These are called the PRINCIPAL VALUES of cos x.



y = cos x

Domain: $[0, \pi]$

Range: [-1, 1]

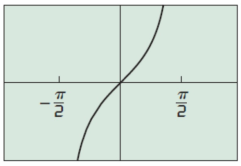


$y = \cos^{-1} x$ or y = arccos x

Domain: [-1, 1]

Range: $[0, \pi]$

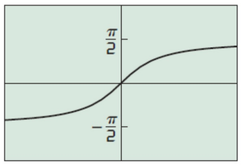
A one-to-one portion of y = tan x can be found on the interval $(-\frac{\pi}{2}, \frac{\pi}{2})$.



y = tan x

Domain: $(-\frac{\pi}{2}, \frac{\pi}{2})$

Range: $(-\infty, \infty)$



$y = \tan^{-1} x$ or y = arctan x

Domain: $(-\infty, \infty)$

Range: $(-\frac{\pi}{2}, \frac{\pi}{2})$

When we evaluate inverse trig functions, we must consider the appropriate domains and ranges.

Give the exact value, in radians.

1. arcsin $(-\frac{1}{2})$
   rewrite: principal values:

2. $\cos^{-1} 1$
   rewrite: principal values:

3. arctan (-1)
   rewrite: principal values:

4. Find arccos(0.445) in degrees.

5. Find $\tan^{-1}(-2.75)$ in radians.

Now let's do some composition of trig functions and inverse trig functions.

6. $\arcsin(\sin\frac{\pi}{6})$

7. $\arcsin(\sin\frac{7\pi}{6})$

**Why doesn't the arcsin just "undo" the sin in #7?

8. $\tan(\sin^{-1}1)$

9. $\sin^{-1}(\cos\frac{3\pi}{4})$

Find an algebraic expression for each.
Hint: Draw a right triangle!

10. $\cos(\arctan x)$

11. $\cot(\cos^{-1} x)$

12. $\sin(\arccos 3x)$