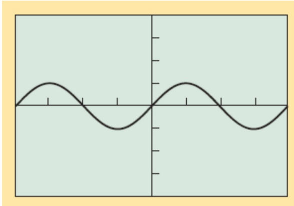
4.4 Graphs of $y = \sin x$ and $y = \cos x$

ESSENTIAL QUESTIONS:

Can you generate the graphs of the sine and cosine functions?

How can the transformations in $y = a \sin(bx + c) + d$ be described?

$y = \sin x$



[–2π, 2π] by [–4, 4]

Domain: _______________

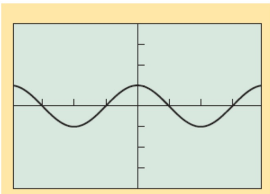
Range: ________________

The graph of $y = \sin x$ is called a periodic function because it repeats itself. The period is the number of units it takes for the graph to complete one full cycle.

period of $y = \sin x$

________° = ________ radians.

$y = \cos x$



[–2π, 2π] by [–4, 4]

Domain: _______________

Range: ________________

The graph of $y = \cos x$ is also periodic. How does its period compare to that of $y = \sin x$?

DEFINITION Sinusoid

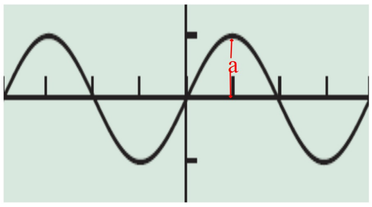
A function is a **sinusoid** if it can be written in the form

$$f(x) = a \sin(bx + c) + d$$

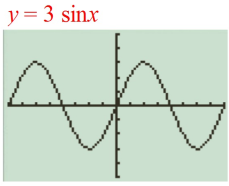
where a, b, c, and d are constants and neither a nor b is 0.

$|a|$ = amplitude

(amplitude is defined the same way for both sine and cosine)



$y = 3 \sin x$



[-2π, 2π] by [-5, 5]

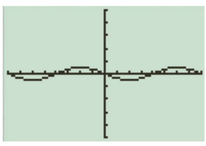
Amplitude:

Domain:

Range:

Period:

$y = -\frac{1}{2} \sin x$



[-2π, 2π] by [-5, 5]

Amplitude:

Domain:

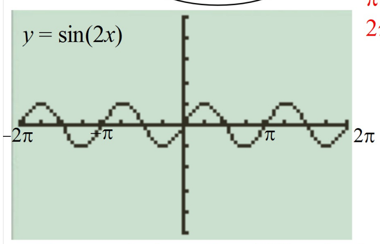
Range:

Period:

The **period** of a sine or cosine curve in the form $y = a \cdot \sin(bx + c) + d$ or $y = a \cdot \cos(bx + c) + d$ is

$$\text{period} = \frac{2\pi}{b}$$

Notice that a complete cycle must now be squished into π units instead of the usual 2π units.

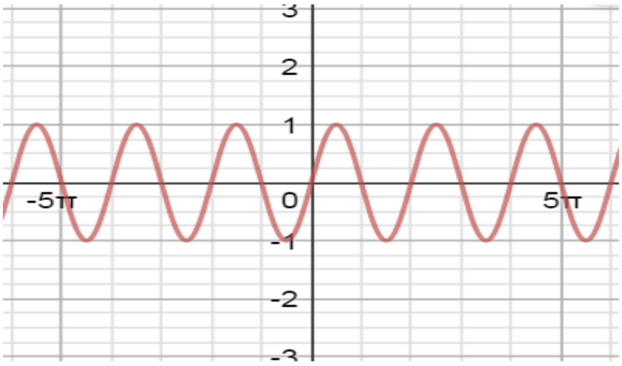


Frequency is the reciprocal of period.

$$\text{frequency} = \frac{b}{2\pi}$$

Frequency is the number of full cycles the wave completes in 1 unit interval. The concept of frequency is important to such things as radio waves, microwaves, and other electrical and engineering applications.

Describe the transformations to y = sinx when the equation becomes $y = -2\sin\left(\frac{x}{3}\right)$. Then graph both equations on the same axes.

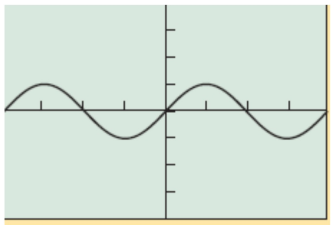


[-6π, 6π] by [-3, 3]

A horizontal shift of a sinusoid is called a **phase shift**. We can find the phase shift of $y = a \cdot \sin(bx + c) + d$ and $y = a \cdot \cos(bx + c) + d$ as follows:

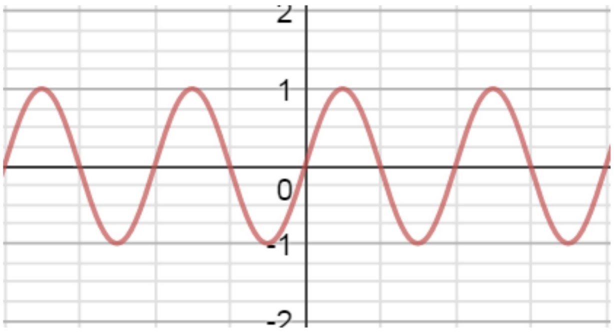
$$\text{phase shift} = \frac{c}{b}$$

Graph y = sin(x - π/2) on the graph below.



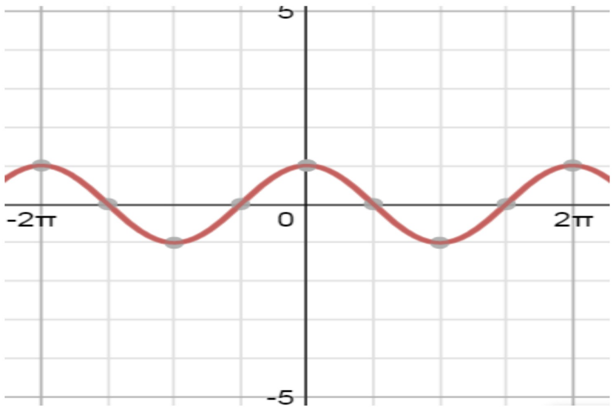
[−2π, 2π] by [−4, 4]

Describe the transformations to the graph of $y = \sin x$ if $y = \sin(2x + \pi/3)$. Then graph.



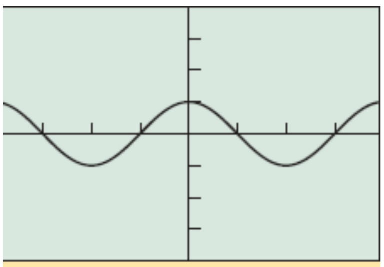
[-4π, 4π] by [-2, 2]

Describe the transformations to the graph of $y = \cos x$ if $y = 4\cos(-2x - \pi)$. Then graph.



A vertical shift of a sinusoid is called vertical displacement. The vertical displacement of $y = a \cdot \sin(bx + c) + d$ and $y = a \cdot \cos(bx + c) + d$ is "d".

Graph $y = \cos x + 2$



[−2π, 2π] by [−4, 4]

Note that the "+ 2" is NOT in parentheses, so it has nothing to do with the phase shift.

Work backwards! Write an equation for a cosine function that has amplitude ¼, period 4π, phase shift π/2, and vertical shift 3.