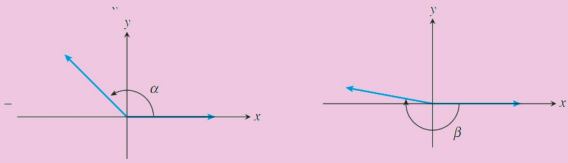
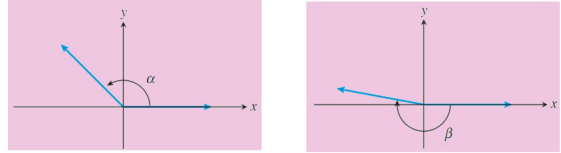


4.3 Circular Trig

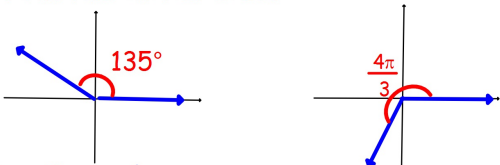


ESSENTIAL QUESTION:
How do you transfer what you know about right triangle trig to problems involving circular trig functions?

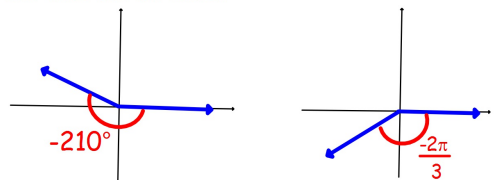
Angles in _____
have their initial side along the _____
and their vertex at the _____.



Positive angles open _____
from the initial side.

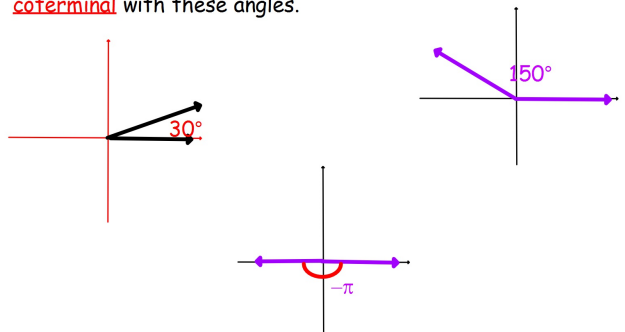


Negative angles open _____
from the initial side.



Two angles can have the same initial and terminal sides but different measures, depending on how many revolutions it takes to get to the terminal side.

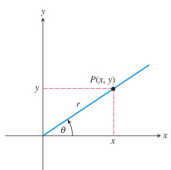
Name a positive angle and a negative angle that are **coterminal** with these angles.



EXPLORATION 1 Investigating First Quadrant Trigonometry

Let $P(x, y)$ be any point in the first quadrant (Q1), and let r be the distance from P to the origin. (See Figure 4.24.)

1. Use the acute angle definition of the sine function (Section 4.2) to prove that $\sin \theta = y/r$.
2. Express $\cos \theta$ in terms of x and r .
3. Express $\tan \theta$ in terms of x and y .
4. Express the remaining three basic trigonometric functions in terms of x , y , and r .



example...

Let θ be the acute angle in standard position whose terminal side contains the point $(5, 3)$. Find the six trigonometric functions of θ .

Let θ be the angle in standard position whose terminal side contains the point $(-5, 12)$. Find the 6 trig functions of θ .

DEFINITION Trigonometric Functions of any Angle

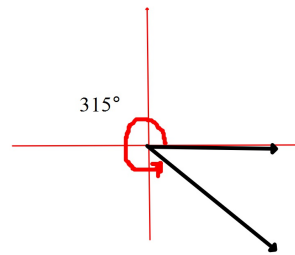
Let θ be any angle in standard position and let $P(x, y)$ be any point on the terminal side of the angle (except the origin). Let r denote the distance from $P(x, y)$ to the origin, i.e., let $r = \sqrt{x^2 + y^2}$. (See Figure 4.25.) Then

$$\sin \theta = \frac{y}{r} \qquad \csc \theta = \frac{r}{y} \ (y \neq 0)$$

$$\cos \theta = \frac{x}{r} \qquad \sec \theta = \frac{r}{x} \ (x \neq 0)$$

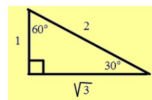
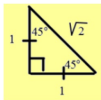
$$\tan \theta = \frac{y}{x} \ (x \neq 0) \qquad \cot \theta = \frac{x}{y} \ (y \neq 0)$$

Find the six trigonometric functions of 315° .



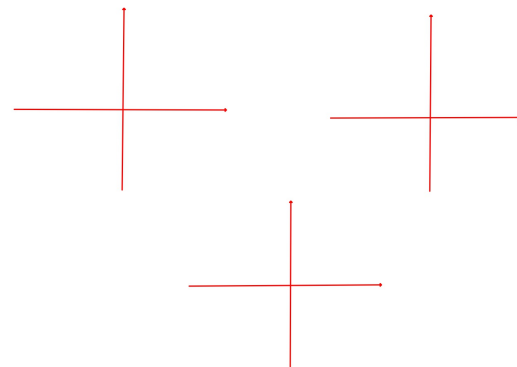
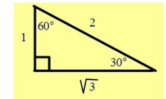
Steps for Evaluating Trig Functions

1. Draw the angle θ in standard position, being careful to place the terminal side in the correct quadrant.
2. Without declaring a scale on either axis, label a point P (other than the origin) on the terminal side of θ .
3. Draw a perpendicular segment from P to the x -axis, determining the *reference triangle*. If this triangle is one of the triangles whose ratios you know, label the sides accordingly. If it is not, then you will need to use your calculator.
4. Use the sides of the triangle to determine the coordinates of point P , making them positive or negative according to the signs of x and y in that particular quadrant.
5. Use the coordinates of point P and the definitions to determine the six trig functions.



Find the following without a calculator:

- (a) $\sin(-210^\circ)$
- (b) $\tan(5\pi/3)$
- (c) $\sec(-3\pi/4)$



★ Finding trig values when the terminal side of the angle is on an axis:

Find each of the following, if it exists. If the value is undefined, write "undefined."

- (a) $\sin(-270^\circ)$
- (b) $\tan 3\pi$
- (c) $\sec \frac{11\pi}{2}$

Sometimes you can use the value of one trig function to find the values of all the others.

Find $\cos \theta$ and $\tan \theta$ by using the given information to construct a reference triangle.

- (a) $\sin \theta = \frac{3}{7}$ and $\tan \theta < 0$
- (b) $\sec \theta = 3$ and $\sin \theta > 0$
- (c) $\cot \theta$ is undefined and $\sec \theta$ is negative