4.3 Use Integrals to Express Area

ESSENTIAL QUESTION

How can the area of a plane region be expressed as a definite integral or the sum of 2 or more definite integrals?

We can use integrals to write expressions that represent the area of a region bounded by the graph of f(x), the x-axis, and the lines x = a and x = b.

$$A = \int_a^b f(x)dx$$

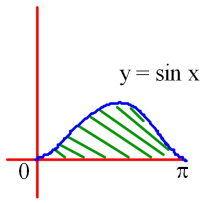
This expression is called a definite integral.

Notice how the definite integral is similar to the limit definition of area of a plane region.

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\cdot \Delta x \quad \Leftrightarrow \quad A = \int_a^b f(x)dx$$

Write an integral that represents the area of each region.

1.



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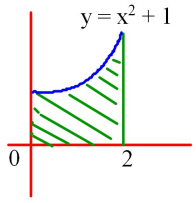

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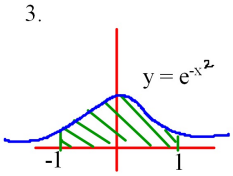

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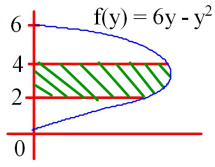


Properties of Definite Integrals

1. $\int_a^b f(x)dx = -\int_b^a f(x)dx$

2. $\int_a^a f(x)dx = 0$

3. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

   if $a \le c \le b$.

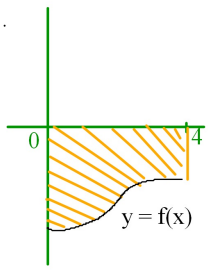
Areas below the x-axis

The area of a region should be positive. However, the value of a definite integral that lies below the x-axis will be negative.

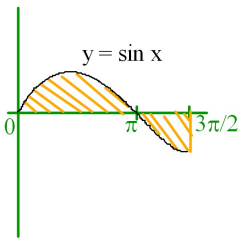
How can we use the properties of definite integrals to create areas that give positive values?

Write a definite integral that could be used to express the area of the shaded region.

1.



2.



Graph the region bounded by $y = |x^2 - 4x + 3|$. Then write an integral expression that represents the area of the regions bounded by this curve, the x-axis, and the lines $x = 0$, and $x = 5$.