4.2 More Riemann Sums
4.3 Expressing Area as a Definite Integral

ESSENTIAL QUESTIONS

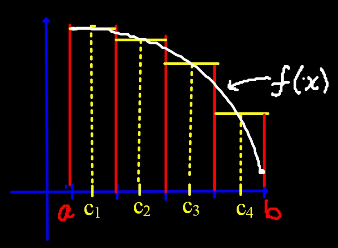
1. Why can the Midpoint Rule provide a more accurate estimate for area than a lower sum or upper sum?
2. How is the limit def of area the same as expressing area as a definite integral?

Last class, we learned how to estimate the area of a plane region by finding areas of rectangles and summing these areas.

This method is called
Riemann Sums

Riemann Sums come in many forms--upper sums, lower sums, midpoint sums, and the limit of sums.

Midpoint Rule: Let c_i be the midpoint of each partition of an interval [a, b]. Use $f(c_i)$ as the height of each rectangle instead of the left side or the right side of the rectangle.



Example: Use the midpoint rule to find the area of the region bounded by $y = 4x - x^2$ and the x-axis on [0, 4]. Let n = 4.

We know:

1. The actual area under a curve lies between the lower sum and the upper sum.

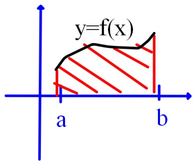
   $s(n) \leq A \leq S(n)$

2. We can estimate the area more accurately by making more rectangles; i.e. by making $n \to \infty$ and $\Delta x \to 0$.

Limit Definition of Area

$$A = \lim_{n \to \infty} \sum_{i=1}^{n} f(c_i) \cdot \Delta x \quad \text{where} \quad \Delta x = \frac{b-a}{n}$$

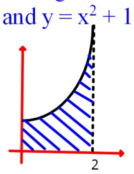
and $c_i = a + i \cdot \Delta x$



Interpretation: As the number of rectangles approaches infinity, the sum of their areas ***exactly*** equals the area under the curve.

Example: Use the limit definition of area to find the area of the region enclosed by the graphs of $x = 0$, $x = 2$, the x-axis, and $y = x^2 + 1$.

Recall that last class, we approximated this area with upper & lower sums, with n = 4.



Example: Use the limit definition of area to find the area of the region enclosed by the graphs of $x = 0$, $x = \pi$, the x-axis, and $y = \sin x$.