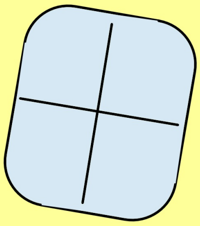
On your white board, graph $y = x^2 + 1$ for $-4 \leq x \leq 4$.

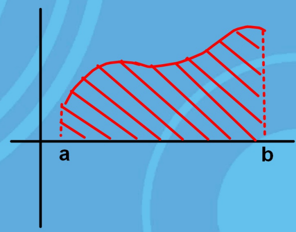


Using any means you can think of, approximate the area of the region enclosed by $y = x^2 + 1$, the x-axis, and the lines $x = -4$ and $x = 4$.

4.2 Upper & Lower Sums

ESSENTIAL QUESTION

How can we estimate the area of a plane region under a curve using upper and lower sums?



Review Sigma Notation

aka Summation Notation

$$\sum_{n=1}^{3}(2n-3)=$$

$$\sum_{n=1}^{5}4=$$

In general, if k is a constant,

$$\sum_{i=1}^{n} ka_i = k\sum_{i=1}^{n} a_i$$

and

$$\sum_{i=1}^{n}(a_i \pm b_1) = \sum_{i=1}^{n} a_i \pm \sum_{i=1}^{n} b_i$$

Special Sums

Memorize these!

$$\sum_{i=1}^{n} k = kn$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

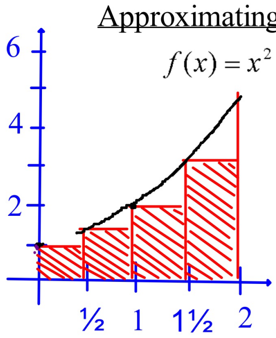
Examples:

1. $\sum_{i=1}^{n}\frac{i+1}{n^2}=$

2. $\lim_{n\to\infty}\sum_{i=1}^{n}\left(1+\frac{2i}{n}\right)^2\left(\frac{2}{n}\right)=$

Approximating Area Under a Curve

$f(x)=x^2+1$



Area = ht · width
height = $f(x_i)$
width = Δx

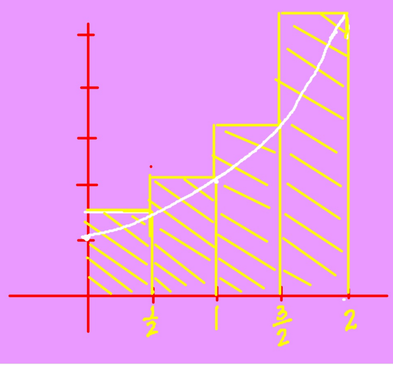
For this problem, using the left side of each rectangle as the height gives us a lower sum.

$$A\approx f(0)\cdot\frac{1}{2}+f\left(\frac{1}{2}\right)\cdot\frac{1}{2}+f(1)\cdot\frac{1}{2}+f\left(\frac{3}{2}\right)\cdot\frac{1}{2}$$

$$A\approx f(0)\cdot\frac{1}{2}+f\left(\frac{1}{2}\right)\cdot\frac{1}{2}+f(1)\cdot\frac{1}{2}+f\left(\frac{3}{2}\right)\cdot\frac{1}{2}$$

The lower sum underapproximates the area.
How could we make the area more accurate?

We could have chosen the right side of each rectangle as the height, and this would have given us the upper sum.



The upper sum overapproximates the area of the region under the curve.

lower sum < actual area < upper sum

Example: Estimate the area of the region bounded by the graph of y = cos x, $0\le x\le\pi/2$, and the x-axis using upper and lower sums. Use 4 partitions.