3.4 Concavity & the Second Derivative Test

ESSENTIAL QUESTIONS

1. How is the sign of the 2nd deriv used to determine where a graph is concave up or down?
2. How is the 2nd deriv used to test for relative max/min?

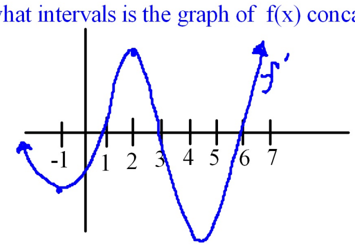
Two ways to determine the concavity of a graph:

1. ***Use the 1st derivative***:

   a) If f '(x) is increasing, then f(x) is concave up.

   b) If f '(x) is decreasing, then f(x) is concave down.

On what intervals is the graph of f(x) concave up?



2. ***Use the 2nd derivative***:

   a) If f "(x) > 0, then f(x) is concave up.

   b) If f "(x) < 0, then f(x) is concave down.

On what intervals is $f(x) = 2x^3 - 3x^2 - 12x$ concave down?

A point of inflection occurs where the graph of *f* changes concavity.

** If (c, f(c)) is a point of inflection, then either f "(c) = 0 or f "(c) is undefined.

The converse of this statement may not be true. Just because f "(c) = 0 or f "(x) is undefined doesn't guarantee a point of inflection at x = c.

Example: Find the points of inflection for y = x - cos x on the interval (0, 2π).

2nd Derivative Test for Relative Max/Min

Let f be a function such that f '(c) = 0 (c is a critical number from the 1st derivative) and the 2nd derivative exists on an open interval containing c.

1) If f "(c) > 0, then f(c) is a relative minimum.

2) If f "(c) < 0, then f(c) is a relative maximum.

3) If f "(c) = 0, the test fails. Use the 1st derivative test instead.

Example: Use the 2nd derivative test to find the relative extrema of $f(x) = \sin x + \cos x$ on $(0, 2\pi)$.

Example: Find the extrema and points of inflection for $f(x) = x^3 - 9x^2 + 27$.

Example: Sketch the graph of a function having these characteristics:

$f(0) = f(2) = 0$

$f'(x) > 0$ if $x < 1$

$f'(1) = 0$

$f'(x) < 0$ if $x > 1$

$f''(x) < 0$