3.3 Increasing & Decreasing Functions and the First Derivative Test

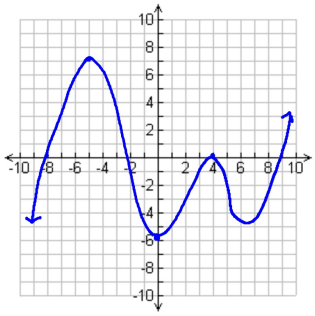
ESSENTIAL QUESTIONS

1. How can the first derivative be used to determine the intervals on which a function is increasing or decreasing?

2. How can the first derivative be used to determine the relative extrema of a function?

A function is increasing on an interval if, as x increases, f(x) increases.

A function is decreasing on an interval if, as x increases, f(x) decreases.



On what interval(s) is f(x) increasing?

On what interval(s) is f(x) decreasing?

Test for Increasing or Decreasing Functions

Let f(x) be continuous on [a, b] and differentiable on (a, b).

1. $f'(x) > 0$ implies f(x) is increasing on [a, b].
2. $f'(x) < 0$ implies f(x) is decreasing on [a, b].
3. $f'(x) = 0$ implies f(x) is constant on [a, b].

To find intervals on which f(x) is increasing or decreasing:

1. Find the critical numbers (where $f'(x) = 0$ or $f'(x)$ is undefined.)
2. Determine the sign of $f'(x)$ in the intervals between the critical numbers.
3. Use the test for increasing/decreasing functions.

Example: Find the intervals on which $f(x) = x^3 - 3x + 2$ is increasing and/or decreasing. Justify your answer.

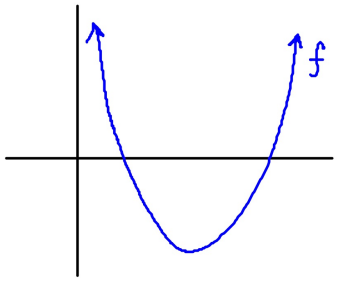
First Derivative Test:

Let c be a critical number of f on (a, b). Then

1. $f(c)$ is a relative maximum if $f'(x)$ changes from positive to negative on either side of c.
2. $f(c)$ is a relative minimum if $f'(x)$ changes from negative to positive on either side of c.

Example: Find the relative extrema for f(x) = sinx cosx is on the interval $(0, 2\pi)$. Justify your answer

The graph of a function f is given below. Sketch the graph of f' on the sames set of axes.



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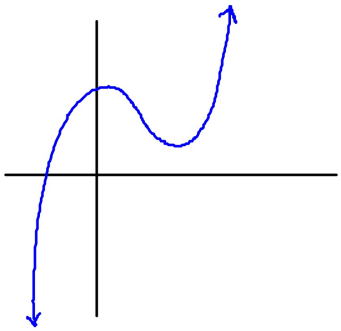
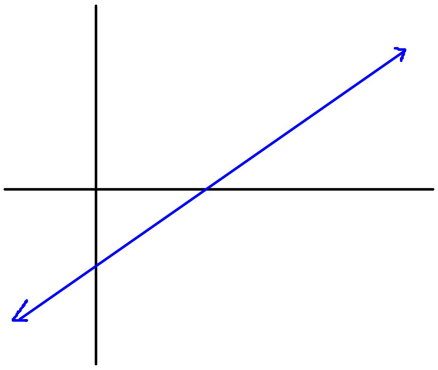


Now reverse the process! The graph of a function f' is given below. Sketch the graph of f on the sames set of axes.



The graph of a function f' is given below. Sketch the graph of f on the sames set of axes.