3.2 Rolle's Theorem & Mean Value Theorem for Derivatives

ESSENTIAL QUESTIONS

What is the significance of Rolle's Theorem and the Mean Value Theorem for Derivatives?

From our last lesson we learned

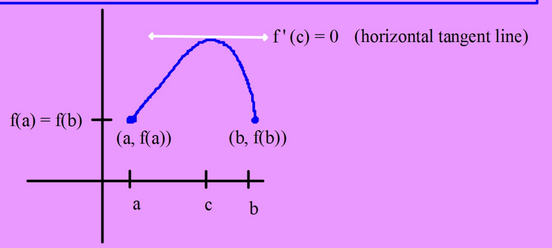
Extreme Value Theorem

If f(x) is continuous on [a, b], then it has both a maximum and a minimum on [a, b].

This is called an existence theorem. It doesn't tell us what values the max and min have, only that they exist.

Rolle's Theorem:

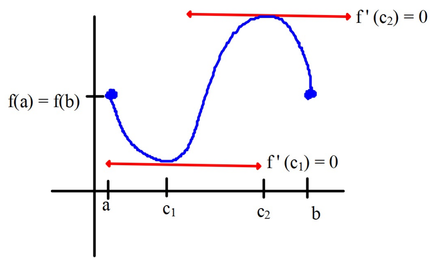
Let f (x) be continuous on [a, b] and differentiable on (a, b). If $f(a) = f(b)$, then there is at least one c in (a, b) such that $f'(c) = 0$.



Remember that f ' (c) = 0 implies that c is a critical number!

Notice that 3 conditions must be met for Rolle's Theorem to apply:

1. f(x) must be continuous on [a, b].
2. f(x) must be differentiable on (a, b).
3. f(a) = f(b)



Examples: Determine whether Rolle's Theorem can be applied to each of the following.

$$f(x) = \frac{x^2 - 1}{x} \quad \text{on } [-1, 1]$$

Does Rolle's Theorem apply? If so, find the value(s) of c where f '(c) = 0.

$$f(x) = \cos x \quad \text{on } [0, 2\pi]$$

Mean Value Theorem for Derivatives

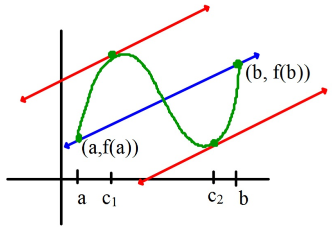
Let f(x) be continuous on [a, b] and differentiable on (a, b). Then there exists a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

slope of tangent line

slope of secant line

Here's a graphical interpretation:



slope of tangent line = slope of secant line

$$f'(c_1) = \frac{f(b) - f(a)}{b - a}$$

$$f'(c_2) = \frac{f(b) - f(a)}{b - a}$$

STEPS FOR APPLYING THE MVT:

1. State whether f(x) is continuous on the given interval.
2. Find f ' (x).
3. Find $\frac{f(b) - f(a)}{b - a}$.
4. Set the expressions in #2 & #3 equal and solve for x.
5. The x values in the interval (a, b) are the c's we're looking for.

Examples:

Apply the MVT to $f(x) = x^3 - x^2 - 2x$ on [-1, 1] to find all values of *c* on (-1, 1) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$

a) Apply the MVT to $f(x) = x - 2\sin x$ on [-π, π] to find all values of *c* on (-π, π) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$

b) Write equations for the tangents to f(x) at the values of c that you just found.