3.2 Applications of Exponential & Logistics Functions

ESSENTIAL QUESTION

How are exponential growth & decay, & logistics growth used to model and solve problems?

Exponential Functions: Growth and decay with a constant percentage rate

$$P(t) = P_0(1 + r)^t$$

P_0 is initial amount
r is rate, given as a decimal
t is time, in years

When $r > 0$, we have a growth model.
When $r < 0$, we have a decay model.

examples:

Tell what the growth (or decay) rate is in each example.

1. $P(t) = 7514(.98)^t$
2. $P(t) = 124(1.35)^t$

3. The population of River City was 4200 in 1910. The population increased by 2.25% per year.
   a. Estimate the population in 1930 and 1945.
   b. When did the population reach 20,000?

4. The initial mass of an object is 15 grams, but its mass decreases by 0.9% every hour. What is its mass at the end of one full day?

We often want to look at how long it takes for populations to **double** their size or **halve** their size.

** The time to double a population is related to a fixed fraction of time!

For example, if c grams of a substance doubles every 10 days, then $y = c(2)^{\frac{t}{10}}$

Or if the substance halves its amount every 6 hours, then $y = c\left(\frac{1}{2}\right)^{\frac{t}{6}}$

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5. The number of bacteria in a petri dish doubles every 4 hours. How long will it take for 100 bacteria to increase in size to 50,000 bacteria?

In section 3.1, we looked at radioactive carbon-14, and we found its half-life to be about 5700 years. We used an exponential model with base e: $y = y_0e^{kt}$. We can also do half-life problems with a base of ½.

6. The half-life of radium-226 is 1620 years. If 20 grams of r-226 remain in a sample after 500 years, how many grams were in the initial sample?

Use the data in the table, along with your calculator, to find an exponential regression model for the U.S. population. Predict the population in 2015.

Table 3.9 U.S. Population (in millions)

Year	Population
1900	76.2
1910	92.2
1920	106.0
1930	123.2
1940	132.2
1950	151.3
1960	179.3
1970	203.3
1980	226.5
1990	248.7
2000	281.4
2007	301.6

Source: World Almanac and Book of Facts 2009.

Logistics Models: Maximum sustainable growth

Population of Deer The population of deer after t years in Cedar State Park is modeled by the function

$$P(t) = \frac{1001}{1 + 90e^{-0.2t}}.$$

(a) What was the initial population of deer?

(b) When will the number of deer be 600?

(c) What is the maximum number of deer possible in the park?

Other applications of logistics models:

Watauga High School has 1200 students. Bob, Carol, Ted, and Alice start a rumor, which spreads logistically so that $S(t) = 1200/(1 + 39 \cdot e^{-0.9t})$ models the number of students who have heard the rumor by the end of Day t.

(a) How many students have heard the rumor by the end of Day 0?

(b) How long does it take for 1000 students to hear the rumor?