

3.1 Extrema on an Interval

ESSENTIAL QUESTION

What is the difference between absolute and relative extrema?

Objectives:

1. Understand the definition of extrema on an interval.
2. Understand the def. of relative extrema on an open interval.
3. Find extrema on a closed interval.

A big idea in calculus involves the behavior of a function on a given interval.

We want to know about **maxima** and **minima** of a graph, as well as **intervals** on which the graph is **increasing** or **decreasing**.

We can use derivatives to figure these things out.

Example:

$f(x) = x^2 - 3$ on the interval $[-1, 2]$.

Is f continuous on $[-1, 2]$?

Where is the absolute maximum?

Where is the absolute minimum?

Are there any relative maxs or mins?

If we were to change the interval to $(-1, 2)$, what would change about the maxs and mins?

What if the following condition was included:

$$f(x) = x^2 - 3, x \neq 0?$$



Relative Extrema and Critical Points

Theorem: If $f'(c) = 0$ or if f' does not exist at c , then c is called a **CRITICAL NUMBER** of f .

Note: All relative extrema occur at critical numbers, but not all critical numbers give relative extrema!

2. Find the value of the derivative at each of the relative extrema.

$$f(x) = \frac{9(x^2 - 3)}{x^3}$$

$$y = |x|$$

3. $f(x) = \sin x$

Finding Absolute Extrema on a Closed Interval



Example:

Find the absolute extrema of $f(x) = 3x^4 - 4x^3$ on the interval $[-1, 2]$.