3.1 Exponential & Logistics Functions

ESSENTIAL QUESTIONS

1. What characteristics identify the graphs of exponential & logistics functions?
2. How are exponential & logistics functions used to model real world problems?

An exponential function can be written in the form

$$y = a \cdot b^x$$

where $a \neq 0$, and $b > 0$ but $b \neq 1$.

a = initial value, when x = 0
b = base

Which of the following are exponential functions. Explain your answer.

1. $f(x) = 4 \cdot 2^x$
2. $y = 6x^{-4}$
3. $g(x) = -3(\frac{1}{2})^{2x}$
4. $y = e^{-x}$

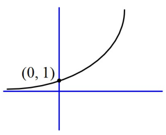
Exponential functions can be used to model growth & decay.

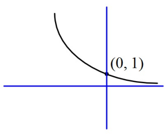
$$y = a \cdot b^x$$

Let $a > 0$.
When $b > 1$, we have exponential growth.
When $0 < b < 1$, we have exponential decay.

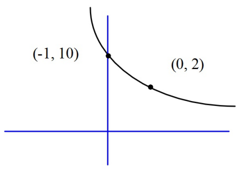
Growth or decay?

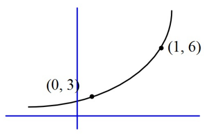
1. $y = 4 \cdot 2^x$
2. $f(x) = (\frac{3}{4})^x$
3. $g(x) = 2 \cdot 3^{-x}$





Write an exponential function given 2 points on the graph. Then tell whether each function represents growth or decay.





Exponential Growth & Decay (using e as the base)

$$y = y_0 \cdot e^{kt}$$

y = ending amount
y_0 = initial amount (when t = 0)
k = constant of variation ($k > 0 \rightarrow$ growth; $k < 0 \rightarrow$ decay)
t = time

The amount (in grams) of radioactive carbon-14 present in a fossil is given by $C = 50e^{-0.0001216t}$.

a. How many grams of C-14 were in the fossil initially?

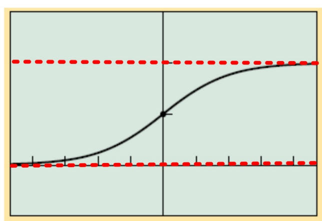
b. How many grams of C-14 are there when t = 1000 years?

c. How many years will it take before only half the initial amount of C-14 remains in the fossil?

Logistics Functions

Very few populations of organisms actually grow exponentially. There is usually some factor in their environment that causes growth to taper off, such as lack of food, predators, or disease.

The logistics function is a much better model for the growth of living things.



Logistics functions take the form

$$y = \frac{c}{1 + a \cdot e^{-kx}}$$

The c in this equation is the upper bound, or limit to growth. In biology, this is called the carrying capacity of the population.

The logistics graph will have 2 horizontal asymptotes, one at $y = c$, and the other at $y = 0$.

Find the y-intercept and the horizontal asymptotes for

$$y = \frac{50}{1 + 2 \cdot e^{-0.015x}}$$

Then sketch a graph.

Based on current census data, the population of Dallas, TX is modeled by

$$P(t) = \frac{1,301,642}{1 + 21.602e^{-0.05054t}}$$

Time t is given in years since 1900. When was the population of Dallas 1,000,000? What is the limit to growth in Dallas? In what year will that number be reached?

***What are the horizontal asymptotes and the y-intercept of the logistics equation

$$f(x) = \frac{75}{25 + 10e^{-0.023t}}$$