

2.5 Implicit Differentiation

OBJECTIVE:

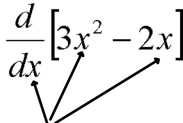
Use the chain rule to differentiate equations that are in implicit form.

Equations come in 2 forms:

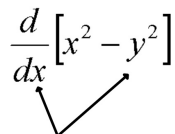
1. **Explicit** - the equation is solved for the dependent variable, like $y = x \sin x$.
2. **Implicit** - the equation is in some form other than explicit, like $x^2y - 4xy^3 + 3x - 2 = 0$. In some cases, it would be very difficult to solve for the dependent variable.

Implicit differentiation is actually a form of the chain rule.

The derivatives that we've done so far have mostly been done with respect to x . When we differentiate terms with variables other than x , we differentiate implicitly.

$$\frac{d}{dx} [3x^2 - 2x]$$


variables agree

$$\frac{d}{dx} [x^2 - y^2]$$


not all variables agree

We'll have to use the chain rule to differentiate the y^2 term.

Steps for Implicit Differentiation:

1. Differentiate both sides of equation with respect to x .
2. Collect all terms containing dy/dx on the left side of the equation.
3. Factor out the dy/dx .
4. Solve for dy/dx .

Examples: Find the derivative of each equation.

1. $x^2 + y^2 = 16$

2. $\frac{d}{dx}[xy^3]=$

3. $\sqrt{xy} = x - 2y$

4. $\cos(x + y) = x + y$

5. Evaluate $\frac{dy}{dx}$ at $\left(2, \frac{\pi}{3}\right)$ for $x \cos y = 1$.

6. Find all points at which the graph of
 $4x^2 + y^2 - 8x + 4y + 4 = 0$
has horizontal or vertical tangents.