2.5 Implicit Differentiation

OBJECTIVE:

Use the chain rule to differentiate equations that are in implicit form.

Equations come in 2 forms:

1. ***Explicit*** **- the equation is solved for the dependent variable, like y = x sinx.**

2. ***Implicit*** **- the equation is in some form other than explicit, like $x^2y - 4xy^3 + 3x - 2 = 0$. In some cases, it would be very difficult to solve for the dependent variable.**

Implicit differentiation is actually a form of the chain rule.

The derivatives that we've done so far have mostly been done with respect to x. When we differentiate terms with variables other than x, we differentiate implicitly.

$$\frac{d}{dx}\left[3x^2 - 2x\right]$$

variables agree

$$\frac{d}{dx}\left[x^2 - y^2\right]$$

not all variables agree

We'll have to use the chain rule to differentiate the y^2 term.

Steps for Implicit Differentiation:

1. **Differentiate both sides of equation with respect to x.**
2. **Collect all terms containing dy/dx on the left side of the equation.**
3. **Factor out the dy/dx.**
4. **Solve for dy/dx.**

Examples: Find the derivative of each equation.

1. $x^2 + y^2 = 16$

2. $\frac{d}{dx}\left[xy^3\right]=$

3. $\sqrt{xy} = x - 2y$

4. $\cos(x+y) = x + y$

5. Evaluate $\frac{dy}{dx}$ at $\left(2, \frac{\pi}{3}\right)$ for $x\cos y = 1$.

6. Find all points at which the graph of

$4x^2 + y^2 - 8x + 4y + 4 = 0$

has horizontal or vertical tangents.