2.4 Real Zeros of Polynomial Functions

Essential Questions

1. How are the Remainder Theorem and the Factor Theorem different?

2. What process can be used to determine the real zeros of a polynomial function?

Long division and synthetic division are techniques that can help us find the rational zeros of polynomial functions.

Review long division: $(x^3 - 3x + 4) \div (x + 2)$

NOTE: Our text calls the result of the division a "summary statement."

Review synthetic division: $(2x^4 + 3x^2 - x + 1) \div (x - 1)$

REMAINDER THEOREM

If a polynomial f(x) is divided by (x - k), then the remainder is f(k).

examples:

1. What is the remainder when $f(x) = 2x^3 - 3x^2 + 4x - 7$ is divided by (x - 2)?

2. What is the remainder when $f(x) = 4x^5 + x^4 - x - 3$ is divided by (x + 4) ?

Factor Theorem

A polynomial f(x) has (x - k) as a factor if and only if f(k) = 0.

example:

Use the Factor Theorem to determine whether x - 3 is a factor of $f(x) = x^3 - x^2 - x - 15$.

Explain how the Remainder Theorem and the Factor Theorem are the same.

RATIONAL ZEROS THEOREM

Let $f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdot \cdot \cdot + a_0$.

If **p** is an integer factor of a_0 and **q** is an integer factor of a_n, then the possible rational factors of f(x) are

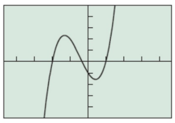
$$x = \pm \frac{p}{q}$$

What do we mean by a RATIONAL zero?

Procedure for finding rational zeros:

1. Use Rational Zeros Theorem to determine possible candidates (p/q) for zeros.
2. Use factor theorem to test candidates. f(x) = 0 means that candidate is a zero.
3. Use synthetic division to find the next polynomial factor.
4. If this factor is a quadratic, either factor or use the quadratic formula to find the remaining zeros. If not, repeat steps 1 - 4 til you find all the zeros.

2. Find the rational zeros of $3x^3 + 4x^2 - 5x - 2$.



Now let's find the REAL zeros of a function, which will include irrational zeros as well as rational zeros.

$f(x) = 2x^4 - 7x^3 - 8x^2 + 14x + 8$