2.3 Product & Quotient Rules for Derivatives

ESSENTIAL QUESTIONS

How are derivatives related to position, velocity, and acceleration functions?

Here are the derivatives for all 6 trig functions:

$f(x) = \sin x \longrightarrow f'(x) = \cos x$

$f(x) = \cos x \longrightarrow f'(x) = -\sin x$

$f(x) = \tan x \longrightarrow f'(x) = \sec^2 x$

$f(x) = \cot x \longrightarrow f'(x) = -\csc^2 x$

$f(x) = \sec x \longrightarrow f'(x) = \sec x \cdot \tan x$

$f(x) = \csc x \longrightarrow f'(x) = -\csc x \cdot \cot x$

Product Rule

If $y = f(x) \cdot g(x)$, then

$$\frac{dy}{dx} = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

Examples: Find the derivatives for these functions.

1. $y = x \sin x$

2. $f(x) = (x^2 - 2x + 1)(x^3 - 1)$

3. $y = \sqrt{x}(x^2 - 1)\sin x$

QUOTIENT RULE

If $y = \dfrac{f(x)}{g(x)}$, $g(x) \neq 0$,

then $$\frac{dy}{dx} = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{[g(x)]^2}$$

Example: Find the derivative of each function.

4. $f(x) = \dfrac{x^2 - 1}{3x - 2}$

5. $\dfrac{d}{dx}\left[\dfrac{5 - \frac{2}{x}}{x + 4}\right]$

6. $y = \frac{\sin x}{4}$

7. $y = \frac{\sin x}{x^2}$

8. $y = \frac{1 + \tan x}{\sec x}$

NOTATION FOR HIGHER ORDER DERIVATIVES

	f(x) =	y =
1st derivative	$f'(x)$ or $\frac{d}{dx}\left[f(x)\right]$	$\frac{dy}{dx}$ or y'
2nd derivative	$f''(x)$ or $\frac{d^2}{dx^2}\left[f(x)\right]$	$\frac{d^2y}{dx^2}$ or y''
3rd derivative	$f'''(x)$ or $\frac{d^3}{dx^3}\left[f(x)\right]$	$\frac{d^3y}{dx^3}$ or y'''
4th derivative	$f^{(4)}(x)$ or $\frac{d^4}{dx^4}\left[f(x)\right]$	$\frac{d^4y}{dx^4}$ or $y^{(4)}$
nth derivative	$f^{(n)}(x)$ or $\frac{d^n}{dx^n}\left[f(x)\right]$	$\frac{d^ny}{dx^n}$ or $y^{(n)}$

Find the first and second derivatives of $y = \sqrt{x}$.

A particle moves along the x-axis. Its motion is described by the function $f(t) = t \sin t$. Find the velocity and acceleration of the particle at $t = \pi/2$.

Note: position is $f(t)$
velocity is $v(t) = f'(t)$
acceleration is $a(t) = v'(t) = f''(t)$