2.3 Higher Order Polynomial Functions

ESSENTIAL QUESTIONS:

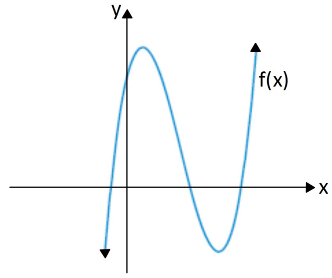
1. How can the leading coefficient and the degree of a polynomial be used to determine its end behavior?

2. How does multiplicity of factors affect the behavior of a graph around its zeros?

3. What is the most common application of the Intermediate Value Theorem?

End behavior of a polynomial function

To discuss the end behavior of a funtion, we look at how the y-values behave as x $\rightarrow\infty$ or x $\rightarrow -\infty$.

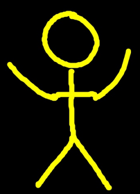


As x $\rightarrow\infty$, f(x) $\rightarrow$______

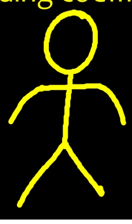
We can also write this in limit notation:

As x $\rightarrow -\infty$, f(x) $\rightarrow$______

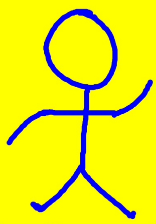
Even degree, leading coefficent is positive



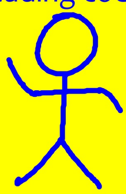
Even degree, leading coefficent is negative



Odd degree, leading coefficient is positive



Odd degree, leading coefficient is negative



THEOREM Local Extrema and Zeros of Polynomial Functions

A polynomial function of degree n has at most $n - 1$ local extrema and at most n zeros.

examples:

Graph the polynomial in a window showing its extrema and zeros and its end behavior. Describe the end behavior using limits.

(a) $f(x) = x^3 + 2x^2 - 11x - 12$

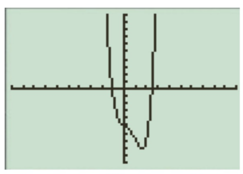
(b) $g(x) = 2x^4 + 2x^3 - 22x^2 - 18x + 35$

Zeros of Polynomial Functions

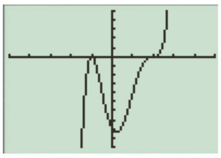
Recall that finding the zeros of a function is the same as finding the x-intercepts of the graph of f(x), or by solving algebraically the equation f(x) = 0.

example....

Use a grapher to find the roots of $f(x) = x^4 - 2x^3 - x - 5$.



Sometimes a functions has repeated zeros, which is also called multiplicity of zeros .



This is the graph of $f(x) = (x - 2)^3(x+1)^2$.

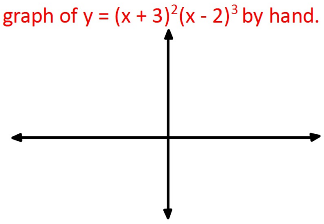
We say x = 2 is a zero of multiplicity 3, and x = -1 is a zero of multiplicity 2.

Notice how the graph just "kisses" the x-axis at x = -1, but it crosses the x-axis at x = 2. We can generalize this behavior by looking at odd and even multiplicities.

ZEROS OF ODD AND EVEN MULTIPLICITY

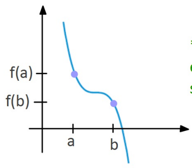
1. If f(x) has a zero of ODD multiplicity at x = c, then f(x) will CROSS the x-axis at x = c.
2. If f(x) has a zero of EVEN multiplicity at x = c, then f(x) will KISS the x-axis at x = c.

Use what you have learned about end behavior and multiplicity to sketch the graph of $y = (x + 3)^2(x - 2)^3$ by hand.



The Intermediate Value Theorem

If f(x) is continuous on the interval [a, b], then f takes on every value between f(a) and f(b).



** This theorem can be used to show the existence of a zero between to numbers if the signs of f(a) and f(b) are different!

example:

Show that $f(x) = x^4 - 2x^3 - x - 5$ has a zero between x = 2 and x = 3.

Modeling with polynomials

You have been given a sheet of cardboard that measures 20 in. by 30 in. You must cut equal squares out of each corner of the cardboard so that when the edges are folded up, you have an open box with volume 550 in^3. How long must the edge of each square be to created such a box, assuming that we want to waste as little cardboard as possible.