2.2 Power Functions

Essential Question

Why is $y = 3x^{1/2}$ a power function, but $y = 3(1/2)^x$ is not?

Power Functions

Any function that can be written in the form

$$f(x) = a \cdot x^b$$

where $a \neq 0$ and $x \neq 0$, is a power function.

a = constant of variation (or proportion)

When the power on x is positive, we have direct variation.
When the power on x is negative, we have inverse variation.

Name	Formula	Power	Constant of Variation
Circumference	$C = 2\pi r$	1	2π
Area of a circle	$A = \pi r^2$	2	π
Force of gravity	$F = k/d^2$	−2	k
Boyle's Law	$V = k/P$	−1	k

A power function passes through the points (1, 0.5) and (3, 13.5). Write an equation for this power function.

Write a power function that models the data in the table below.

x	1	4	9	16	25
y	2	4	6	8	10

Charles's Law The volume of an enclosed gas (at a constant pressure) varies directly as the absolute temperature. If the pressure of a 3.46-L sample of neon gas at a temperature of 302 K is 0.926 atm, what would the volume be at a temperature of 338 K if the pressure does not change?

Modeling Planetary Data with a Power Function

Noted astronomer Johannes Kepler (1571–1630) developed three laws of planetary motion that are used to this day. Kepler's Third Law states that the square of the period of orbit T (the time required for one full revolution around the Sun) for each planet is proportional to the cube of its average distance a from the Sun. Table 2.10 gives the relevant data for the six planets that were known in Kepler's time. The distances are given in millions of kilometers, or gigameters (Gm).

Use the data in Table 2.10 to obtain a power function model for orbital period as a function of average distance from the Sun. Then use the model to predict the orbital period for Neptune, which is 4497 Gm from the Sun on average.

Table 2.10 Average Distances and Orbital Periods for the Six Innermost Planets

Planet	Average Distance from Sun (Gm)	Period of Orbit (days)
Mercury	57.9	88
Venus	108.2	225
Earth	149.6	365.2
Mars	227.9	687
Jupiter	778.3	4332
Saturn	1427	10,760

Source: Shupe, Dorr, Payne, Hunsiker, et al., National Geographic Atlas of the World (rev. 6th ed.). Washington, DC: National Geographic Society, 1992, plate 116.

Modeling Free-Fall Speed Versus Distance

Use the data in Table 2.11 to obtain a power function model for speed p versus distance traveled d. Then use the model to predict the speed of the ball at impact given that impact occurs when $d \approx 1.80$ m.

Table 2.11 Rubber Ball Data from CBR™ Experiment

Distance (m)	Speed (m/s)
0.00000	0.00000
0.04298	0.82372
0.16119	1.71163
0.35148	2.45860
0.59394	3.05209
0.89187	3.74200
1.25557	4.49558

Windmill Power The power P (in watts) produced by a windmill is proportional to the cube of the wind speed v (in mph). If a wind of 10 mph generates 15 watts of power, how much power is generated by winds of 20, 40, and 80 mph? Make a table and explain the pattern.