Basic Rules for Derivatives

ESSENTIAL QUESTION

How are derivatives used to find rates of change?

A function is ***differentiable*** if it has a derivative.

We won't have to use the limit definition after today (boo-hoo)! We're going to be learning some basic rules that will make finding derivatives easier. These rules can be derived from the limit definition.... plus a lot of serious algebra!

BASIC RULES FOR DERIVATIVES

Constant	f(x) = k	f '(x) = 0
Power	$f(x) = x^n$	$f'(x) = n \cdot x^{n-1}$
Constant Multiple	$f(x) = a \cdot x^n$	$f'(x) = an \cdot x^{n-1}$
Sum/Difference	h(x) = f(x) ± g(x)	h '(x) = f '(x) ± g '(x)
Sine	f(x) = sin x	f '(x) = cos x
Cosine	f(x) = cos x	f '(x) = -sin x

Examples for <u>Constant Rule</u>: Find the derivative each function.

1. $f(x) = 3$

2. $y = -\frac{2}{5}$

Examples for <u>Power Rule</u>: Find the derivative of each function.

3. $f(x) = x^3$

4. $y = \sqrt{x}$

5. $y = \frac{1}{x^2}$

Examples for <u>Constant Multiple Rule</u>: Find the derivative of each function.

6. $y = 3x^5$

7. $f(x) = 6\sqrt[3]{x}$

8. $g(x) = \frac{5}{4x^7}$

9. $y = (3x)^2$

Examples for sums and differences: **Find the derivative of each function.**

10. $y = 2x^2 - 3x + 1$

11. $f(x) = -\frac{x^3}{4} + \sin x - \sqrt[4]{x}$

12. $g(x) = (2x-5)(3x+7)$

Examples for sines and cosines: **Find the derivative of each function.**

13. $y = 5\sin x$

14. $g(x) = \frac{\cos x}{3}$

15. $f(x) = 4x - 2\sin x + 3\cos x$

Examples involving slopes and tangent lines:

16. Find the slope of $y = 2x^2 - 3x + 1$ at the point (2, 3).

17. Write an equation for the tangent to y = 2 cos x at the point (π/2, 1).

RATES OF CHANGE

The first derivative can be used to find ***rates of change*** in problems involving:

- Population growth
- Production rates
- Water flow rates
- Velocity

KINEMATICS is the study of the motion of an object along a straight line, either horizontal or vertical, with respect to some point of origin.

In this type of problem, we define s(t) to be a position function. This is a function that gives the displacement of the object from the origin with respect to time.

If distance = rate x time ($d = rt$), then

$$r = \frac{d}{t} = \frac{\text{change in dist}}{\text{change in time}} = \frac{\Delta s}{\Delta t}$$

rate of change

We define ***average velocity*** *(rate)* as

$$\bar{v} = \frac{\Delta s}{\Delta t} = \frac{s(t_2) - s(t_1)}{t_2 - t_1}$$

Example:

Suppose a ball is dropped 100 feet from the top of a building. Its height at time *t* is $s(t) = -16t^2 + 100$ **.**

Find the average velocity of the ball on the time interval [1, 2].

INSTANTANEOUS VELOCITY

Let *s(t)* be the position function for a particle moving along a straight line. The *instantaneous velocity* of *s* is

$$v(t) = s'(t)$$

Note:

*** The average velocity is related to the slope of a secant line through the endpoints of a time interval.**

**** The instaneous velocity is the derivative of the function at a given instant in time.**

The position function for a free-falling body is

$$s(t) = \frac{1}{2}gt^2 + v_0 t + s_0$$

$g = -32\ ft/s^2 = -9.8\ ft/s^2$ (acceleration due to gravity)

$v_0 = initial\ velocity$ (+ if moving up, - if moving down)

$s_0 = initial\ \ position$ (measured from the ground)

Example: A ball is thrown from the top of a 220-foot building at 22 ft/s.
a) What is its velocity at t = 1 second?
b) What is its average velocity from t = 1 sec to t = 3 sec?
c) What is its velocity when it is 112 feet from the ground?