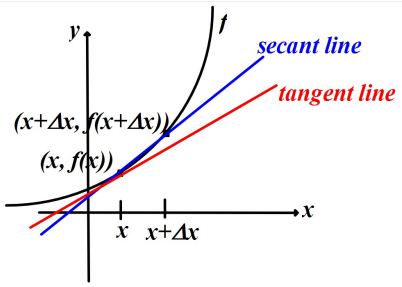
2.1 Limit Definition of Derivative

ESSENTIAL QUESTION

How is the concept of limits used to help us determine a general expression for the slope of a line tangent to a curve?



The slope of the tangent to f can be approximated by the slope of the secant line:

$$m = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

We can make this approximation more accurate by making Δx very small. In fact, as Δx→0,

$$\frac{f(x+\Delta x) - f(x)}{\Delta x} \rightarrow \text{slope of tangent line}$$

Limit Definition of Derivative

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$f'(x)$ **is the derivative of** $f(x)$
(*f* prime of *x*)

What is a derivative?

The derivative of *f (x)* gives a general expression for the slope of a tangent to a curve, and hence the slope of curve itself.

We can evaluate the derivative at any values of *x* and find the slope of the curve at that particular point.

It's important to note that the slope of a line is constant, but the slope of a curve is not.

Examples:

GENERAL EXPRESSION FOR DERIVATIVE FOR ANY *x*.

1. Find the derivative of $f(x) = x^2 - 3$.

SLOPE OF THE TANGENT TO A CURVE AT $x = c$.

2. Find the slope of the tangent to $f(x) = \sqrt{x}$ at $x = 4$.

EQUATION OF A TANGENT LINE
AT A GIVEN POINT ON THE CURVE

3. Find the equation for the tangent line to the graph of $f(x)=\frac{1}{x}$ at the point $\left(\frac{1}{2},\ 2\right)$.

Key words that tell us we'll need to find the derivative of a function:

1. **slope**
2. **tangent line**
3. **rate of change*****

Notation for derivatives

Function	Derivative	How to say it
f(x) =	f ' (x)	derivative of f(x) or f prime of x
f(x) =	$\frac{d}{dx}[f(x)]$	derivative of f(x) with respect to x
y=	$\frac{dy}{dx}$	derivative of y with respect to x
y=	y'	y prime