1.4 Building Functions from Functions
1.5 Inverse Functions

ESSENTIAL QUESTIONS:

1. How can composite functions be used to model real life problems?
2. How are the graphs of a function and its inverse related?

examples of new function notations....

Let $f(x) = \sqrt{x}$ and $g(x) = \sin x$. Find

$(f + g)(x)$

$(fg)(x)$

$\left(\frac{f}{g}\right)(x)$

Now state the domain of each.

DEFINITION Composition of Functions

Let f and g be two functions such that the domain of f intersects the range of g. The **composition f of g**, denoted $f \circ g$, is defined by the rule

$$(f \circ g)(x) = f(g(x)).$$

The domain of $f \circ g$ consists of all x-values in the domain of g that map to $g(x)$-values in the domain of f. (See Figure 1.55.)

$f(x) = x^2$ and $g(x) = x - 3$. Find $(f \circ g)(x)$ and $(g \circ f)(x)$.

$f(x) = \sqrt{x + 2}$ and $g(x) = x^2 - 2x + 1$

Find $f(g(x))$ and $g(f(x))$. Then state the domain of each.

Decomposing Functions

This is the process of "undoing" the composition of functions. Look for a (nested) function inside another function. Call this $g(x)$. The outer function will be $f(x)$.

Examples: For each $h(x)$, decompose $h(x)$ so that $h(x) = f(g(x))$.

1. $h(x) = \sqrt{4 - x}$

2. $h(x) = (\sin x)^2 - 2\sin x + 3$

3. $h(x) = e^{\sqrt{x}}$

RELATIONS & IMPLICITLY DEFINED FUNCTIONS

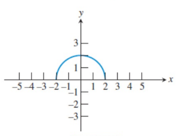
Some curves fail the vertical line test, so we call them relations. For example the circle $x^2 + y^2 = 4$ is a relation. We say this relation is implicitly defined because it is not solved for y.

We can write $x^2 + y^2 = 4$ in explicit form by solving for y, but we actually get 2 explicit functions:

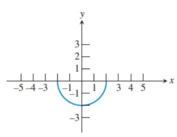
$x^2 + y^2 = 4$

$y^2 = 4 - x^2$

$y = \pm\sqrt{4 - x^2}$



$y = \sqrt{4 - x^2}$



$y = -\sqrt{4 - x^2}$

example.....

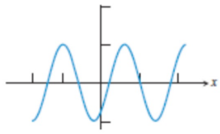
Determine which of the ordered pairs $(2, -5)$, $(1, 3)$, and $(2, 1)$ are in the relation defined by $x^2y + y^2 = 5$. Is the relation a function?

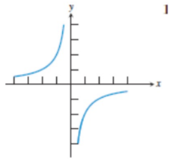
Inverse Functions

A function has an inverse if it is one-to-one.

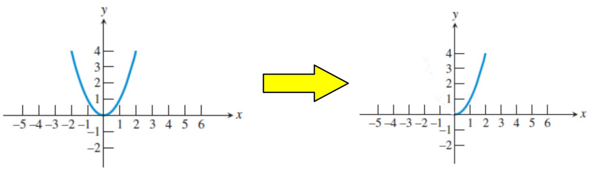
A function is one-to-one if every x in the domain is assigned exactly one y, and every y is assigned exactly one x.

Graphically, a one-to-one function passes both the vertical line test and the horizontal line test.





Sometimes we must restrict the domain of the original function so that we get a one-to-one portion of the curve that contains all range values.



$y = x^2$ $\quad\quad$ $y = x^2$, for $x \geq 0$

Once we have a one-to-one portion of the graph, we can proceed to find the inverse:

Note: The domain of the function becomes the range of the inverse. The range of the function becomes the domain of the inverse.

Given the function $f(x) = \frac{x+3}{x-2}$. Graph.

1. Is f(x) one-to-one?
2. What is the domain and range of f(x)?
3. Find $f^{-1}(x)$.
4. What is the domain and range of $f^{-1}(x)$?

We can prove that two functions f and g are inverses by showing that

f(g(x)) = x and g(f(x)) = x.

example:

If $f(x) = \sqrt{2x - 3}$ and $g(x) = \frac{x^2 + 3}{2}$, prove that f and g are inverses.