1.6 Transformations of Graphs

ESSENTIAL QUESTIONS:

1. What transformations to the graph of $y = f(x)$ are caused by a, b, c, and d when $y = a \cdot f(bx - c) + d$?
2. What happens if a and/or b is negative?

Transforming Graphs in the Coordinate Plane

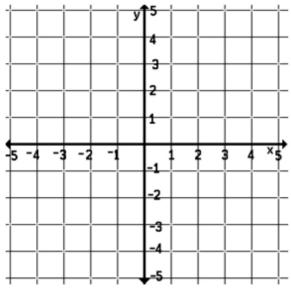
Horizontal (phase) shifts

Let c be positive.

$y = f(x + c)$ → shift f(x) left **c** units

$y = f(x - c)$ → shift f(x) right **c** units

Graph $y = \sqrt{x - 2}$

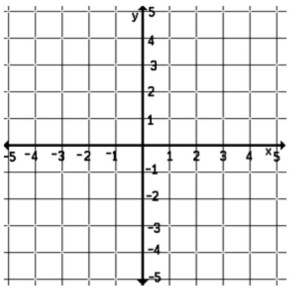


Vertical shifts

$y = f(x) + d$ → shift f(x) up d units

$y = f(x) - d$ → shift f(x) down d units

Graph $f(x) = (x+1)^3 - 2$

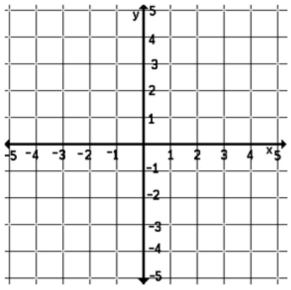


Reflections

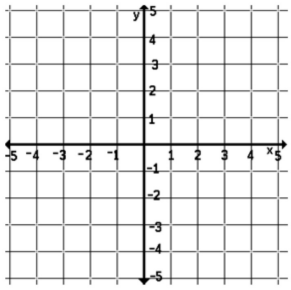
$y = -f(x)$ → reflect in the x-axis

$y = f(-x)$ → reflect in the y-axis

Graph $y = -\ln x$

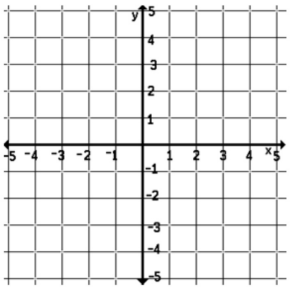


Graph $y = \sqrt{-x}$

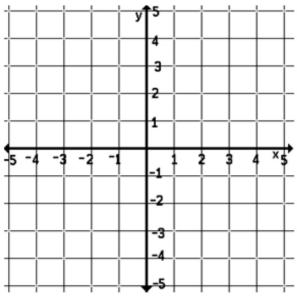


More examples:

1. Graph $y = \text{int}(x)$



2. Graph $y = 2 \cdot \text{int}(x)$



Vertical stretches or shrinks

$y = c \cdot f(x)$

$c > 1$, vertical ***stretch*** by a factor of c

$0 < c < 1$, vertical ***shrink*** by a factor of c

What really happens: Multiply y-coordinates by c, leave the x's alone.

Horizontal stretches or shrinks

$y = f(cx)$

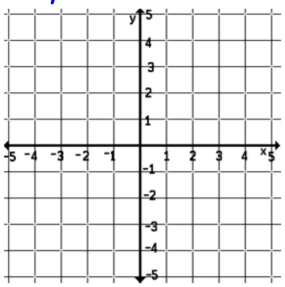
$c > 1$, horizontal shrink by a factor of 1/c

$0 < c < 1$, horizontal stretch by a factor of 1/c

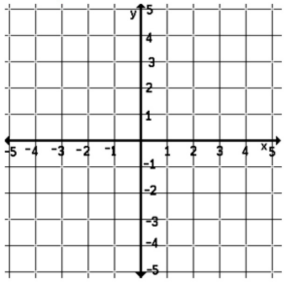
What really happens: Multiply x-coordinates by the reciprocal of c, leave the y's alone.

Graph:

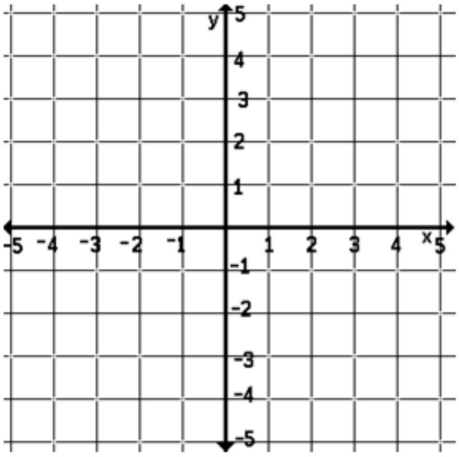
1. $y = 2x^2$



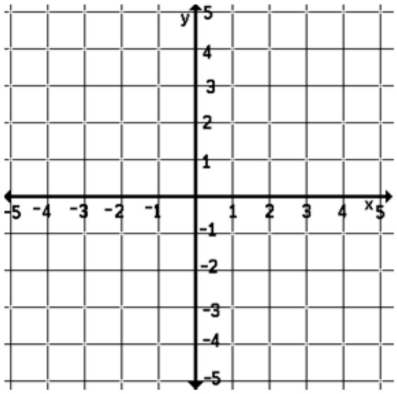
2. $y = (2x)^2$



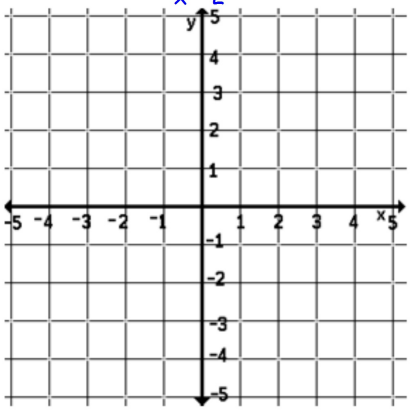
3. $f(x) = 2|x+1| - 3$



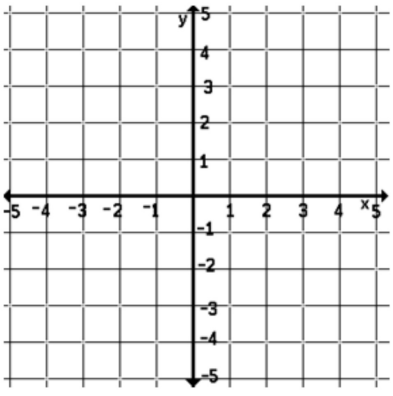
4. $y = e^{-x} - 4$



5. $f(x) = \frac{1}{x-2}$



6. $y = \sqrt{\frac{x}{2}}$



It's easy to graph the absolute value of f(x). Any part of the graph of f(x) that lies below the x-axis will be reflected over the x-axis.

***Why do you think we do this????