1.2 Functions & Their Properties

ESSENTIAL QUESTION:

Why is it important to use graphical, numerical, and algebraic approaches to determine the domain and range of a function?

Domain of a function

For what values of the independent variable is the function defined?

Find the domain. Write your answer in interval notation.

1. $f(x) = \sqrt{x+3}$

2. $f(x) = \dfrac{\sqrt{x}}{x-5}$

3. $A(s) = \dfrac{\sqrt{3}}{4}s^2$

4. $y = \sqrt{16 - x^2}$

Range of a function

Given the domain of a function, what are the possible values of the dependent variable y?

Find the range. Confirm graphically. Write your answer in interval notation.

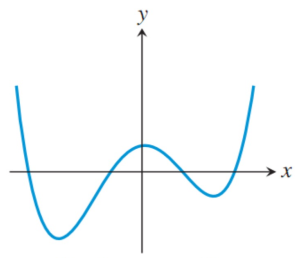
1. $f(x) = \dfrac{4}{x-2}$

2. $y = \sqrt{16 - x^2}$

3. $y = e^x$

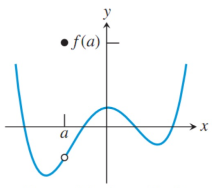
Continuity of a Function

A function is continuous if it has no breaks.

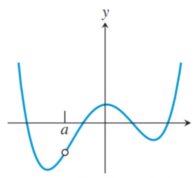


Continuous at all x

Removable Discontinuities (holes)

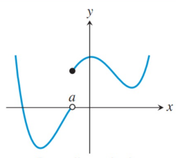


Removable discontinuity

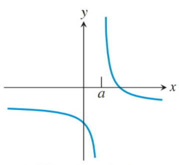


Removable discontinuity

Nonremovable Discontinuites

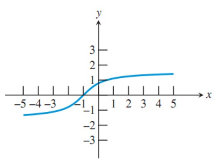


Jump discontinuity

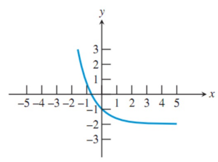


Infinite discontinuity

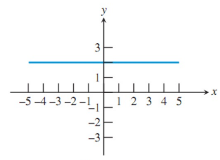
Increasing and Decreasing Functions



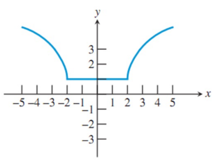
Increasing



Decreasing



Constant



Decreasing on $(-\infty, -2]$
Constant on $[-2, 2]$
Increasing on $[2, \infty)$

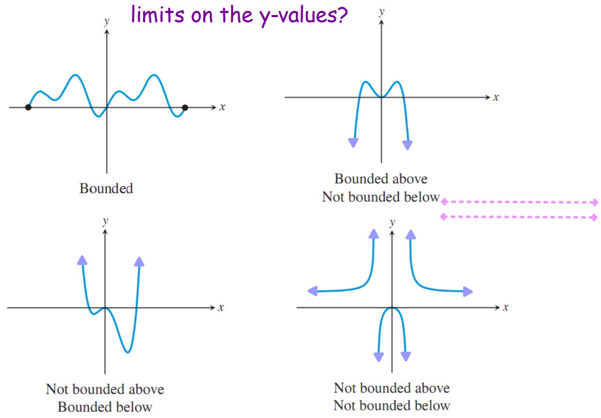
On what interval(s) is each function increasing? Decreasing?

1. $y = (x+3)^3$

2. $f(x) = \dfrac{x^2}{x-1}$

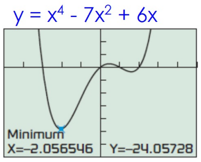
Boundedness

Are there upper and/or lower limits on the y-values?



Local & Absolute Extrema

$y = x^4 - 7x^2 + 6x$



How many local maximums does this graph have?

How many local minimums?

Is there an absolute max or min?

Use your calculator to find the values of the remaining extrema.

Remember: "Extrema" refers to the **y-values** of a function.

Graph $y = x^3 - 4x + 1$. Find all local maxima and minima and the x-values where they occur.

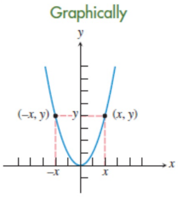
SYMMETRY

1. Symmetry with respect to the y-axis.
   These functions are EVEN functions.

$(x, y) \rightarrow (-x, y)$ and $f(-x) = f(x)$.

Example: $f(x) = x^2$

Graphically



Numerically

x	$f(x)$
−3	9
−2	4
−1	1
1	1
2	4
3	9

Algebraically

$f(x) = x^2$

$f(-x) = (-x)^2 = x^2$

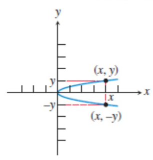
so $f(-x) = f(x)$

2. Symmetry with respect to the x-axis.
   These relations are NEITHER even nor odd.

$(x, y) \rightarrow (x, -y)$

Example: $x = y^2$

Graphically



Numerically

x	y
9	−3
4	−2
1	−1
1	1
4	2
9	3

Algebraically

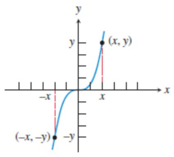
Not functions!

3. Symmetry with respect to the origin.
These functions are ODD.

(x, y) → (-x, -y) and f(-x) = -f(x)

Example: $f(x) = x^3$

Graphically



Numerically

x	y
−3	−27
−2	−8
−1	−1
1	1
2	8
3	27

Algebraically

$f(x) = x^3$
$f(-x) = (-x)^3 = -x^3$
$-f(x) = -x^3$
so $f(-x) = -f(x)$

Tell whether each of the following is even, odd, or neither.

1. $g(x) = \frac{3}{1+x^2}$

2. $f(x) = x^3 + 0.04x + 3$

3. $h(x) = \frac{1}{x}$

Asymptotes

1. Vertical asymptotes occur when the denominator equals 0, so set denominator = 0, cancel any common factors, then find values of x that make the remaining factors = 0.

$$f(x) = \frac{x}{x^3 - 16x}$$

2. Horizontal asymptotes are in the form y = k.

If the degree of the numerator is **less than** the degree of the denominator, then the horizontal asymptote is **y = 0**.

If the degree of the numerator is **greater than** the degree of the denominator, then there is **NO** horizontal asymptote.

If the degree of the numerator is **equal to** the degree of the denominator, then the horizontal asymptote is y = the quotient of the leading coefficients.

Find any vertical or horizontal asymptotes.

1. $f(x) = \frac{x^3 - 1}{x}$

2. $h(x) = \frac{4}{1+x^2}$

End Behavior

As x → ±∞ at the "ends" of the x-axis, what is the behavior of y?

Investigate graphically the end behavior of each function.

1. $h(x) = \frac{1}{x}$

2. $f(x) = e^x$

3. $g(x) = x^3 - 2x^2 + x + 3$