WARMUP:

Solve without a calculator.

1. $x^2 - 3x - 3 = 15$

2. $8^{3x+2} = 4^{5x+1}$

3. $\log x + \log(x-3) = 1$

4. $\sin 2x = \cos x, \quad 0 \le x \le 2\pi$

Investigate.....

1. Is $\lim_{x\to 0} \frac{\sin x}{x} = \lim_{x\to 0} \frac{x}{\sin x} = 1$?

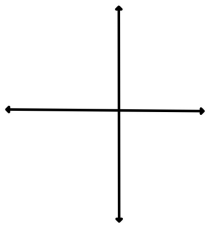
2. Is $\lim_{x\to 0} \frac{1 - \cos x}{x} = \lim_{x\to 0} \frac{x}{1 - \cos x} = 0$?

1-5 Infinite Limits
3-5 Limits at Infinity

Essential Questions

- **What type of abberant behavior in functions is caused by asymptotes and unboundedness?**
- **What is the significance of $y = \lim_{x\to\pm\infty} f(x)$**

Graph $f(x) = \frac{1}{x+3}$



$\lim_{x\to -3^-} f(x) =$

$\lim_{x\to -3^+} f(x) =$

NOTE:

If f(x)→∞ or f(x)→-∞ as x→c, the LIMIT DOES NOT EXIST !
In addition, there will be a vertical asymptote at x = c.

(∞ and -∞ simply describe the unbounded behavior)

EXAMPLES:

$\lim_{x\to -1/2^+} \frac{x^2 - 4}{2x^2 + 5x + 2} =$

$\lim_{x\to -2} \frac{x^2 - 4}{2x^2 + 5x + 2} =$

$$\lim_{x\to 1^+}\frac{2+x}{1-x}=$$

$$\lim_{x\to 4^-}\frac{x^2}{x^2-16}=$$

$$\lim_{x\to \pi/2^+}\frac{-2}{\cos x}=$$

Limits at infinity

When x→∞ (or -∞), we look at the end behavior of the graph.

Definition:

The line $y = L$ is the horizontal asymptote of $f(x)$ if $\lim_{x\to-\infty} f(x)=L$ and $\lim_{x\to\infty} f(x)=L$.

Rules for limits at ±∞

1. $\lim_{x\to\infty}\frac{k}{x^n}=0$ k is a constant; n > 0

If f(x) and g(x) are polynomials:

2. $\lim_{x\to\infty}\frac{f(x)}{g(x)}=0$ if the degree of the numerator is less than the degree of the denominator.

3. $\lim_{x\to\infty}\frac{f(x)}{g(x)}=\infty$ if the degree of the numerator is greater than the degree of the denominator.

4. $\lim_{x\to\infty}\frac{f(x)}{g(x)}=$ the quotient of the leading coefficients of the numerator and denominator.

Examples:

1. $\lim_{x\to\infty}\frac{2x^2-3x+1}{x-5}$

2. $\lim_{x\to\infty}\frac{x^2+1}{3x^2-x+7}$

3. $\lim_{x\to-\infty}\frac{x}{x^3-8}$

4. $\lim_{x\to\infty}\cos x$

5. $\lim_{x\to\infty}\frac{\sin x}{x}$

Functions that have 2 horizontal asymptotes:

$$f(x)=\frac{3x-2}{\sqrt{2x^2+1}}$$

a) Determine $\lim_{x\to\infty}\frac{3x-2}{\sqrt{2x^2+1}}$

b) Determine $\lim_{x\to-\infty}\frac{3x-2}{\sqrt{2x^2+1}}$