1-4: Limits & Continuity

OBJECTIVES

In this lesson, we will discuss the continuity of a function, evaluate one-sided limits, and use the Intermediate Value Theorem to prove the existence of a zero of a function in a given interval.

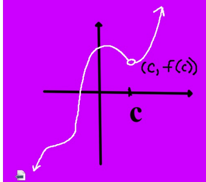
Definition of continuity

A function f is continuous at a point c if these 3 conditions are met:

1. $f(c)$ is defined.
2. $\lim_{x \to c} f(x)$ exists.
3. $\lim_{x \to c} f(x) = f(c)$

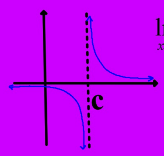
Types of discontinuities:

1. HOLE



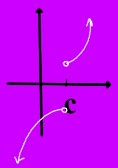
$\lim_{x \to c} f(x) =$

2. ASYMPTOTE



$\lim_{x \to c} f(x) =$

3. JUMP DISCONTINUITY



$\lim_{x \to c} f(x) =$

These discontinuities will be either removable or nonremovable.

REMOVABLE:

✶ occurs in functions with common factors and in some piecewise functions.

✶ to "remove" the discontinuity, redefine the function, usually by factoring.

Examples:

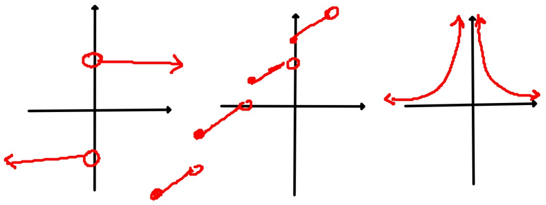
1. $\lim_{x \to -2} \dfrac{2x^2 + x - 6}{x + 2}$

2. **Find a such that f is continuous for all x.**

$$f(x) = \begin{cases} \dfrac{4\sin x}{x}, & x < 0 \\ a - 2x, & x \geq 0 \end{cases}$$

NONREMOVABLE DISCONTINUITIES

- **occur around asymptotes, jump discontinuities, greatest integer functions and other step functions, and some piecewise functions.**



ONE-SIDED LIMITS

$\lim_{x\to c^-} f(x) = L$ ⇨ ***x*** **approaches** ***c*** **from the left.**

$\lim_{x\to c^+} f(x) = L$ ⇨ ***x*** **approaches** ***c*** **from the right.**

It's important to note that

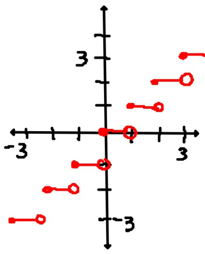
$\lim_{x\to c} f(x)$ **exists**

if and only if

$$\lim_{x\to c^+} f(x) = \lim_{x\to c^-} f(x)$$

Consider the graph of $f(x) = [x]$

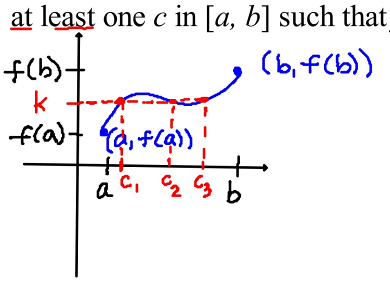
Evaluate the following:



1. $\lim_{x\to 2^+}[x]=$
2. $\lim_{x\to 1.5^-}[x]=$
3. $\lim_{x\to 1.5^+}[x]=$
4. $\lim_{x\to 2^-}[x]=$
5. $\lim_{x\to 2}[x]=$
6. $\lim_{x\to 1.5}[x]=$

INTERMEDIATE VALUE THEOREM

If f is continuous on $[a, b]$ and k is between $f(a)$ and $f(b)$, then there exists at least one c in $[a, b]$ such that $f(c) = k$.



The IVT can be used to prove the existence of the zero of a function in a closed interval.

Example.

Use the IVT to show that $f(x) = 2x^3 - x + 5$ has a zero in the interval [-2, -1].