1.3 Determine Limits Analytically

Essential Question:

What are the 5 analytical techniques used to find the value of a limit?

5 Techniques for looking at limits analytically:
(Always approach the limit in this order!)

1. Direct substitution
2. Factor to rename the function
3. Rationalize the numerator
4. Use special trig limits
5. Use the Squeeze Theorem (rarely!)

Direct substitution: Always try to substitute x into the function first to see if a limit exists.

EXAMPLES:

1. $\lim_{x \to 2} \frac{3x^2 - x + 2}{x + 2}$

2. $\lim_{x \to 64} \sqrt[3]{x}$

Some functions can be simplified by factoring. Then direct substitution can be used on the renamed function. Remember $a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$

3. $\lim_{x \to 2} \frac{x^3 - 8}{x - 2}$

4. $\lim_{x \to 1} \frac{2x^2 - 5x + 3}{x^2 - 1}$

Rationalizing the numerator of an expression can eliminate the problem in the denominator

5. $\lim_{x\to 3}\frac{\sqrt{x+1}-2}{x-3}$

6. $\lim_{x\to 0}\frac{\frac{1}{x+4}-\frac{1}{4}}{x}$

There are two special trig limits that can be used and manipulated to evaluate limits involving trig.

Memorize!

$$\lim_{x\to 0}\frac{\sin x}{x}=1$$

$$\lim_{x\to 0}\frac{\cos x-1}{x}=\lim_{x\to 0}\frac{1-\cos x}{x}=0$$

EXAMPLES for trig limits:

7. $\lim_{x\to 0}\frac{\tan x}{x}$

8. $\lim_{x\to 0}\frac{\sin 5x}{3x}$

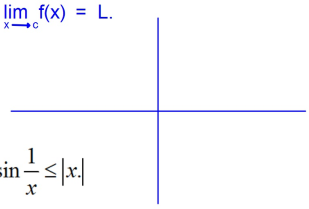
9. $\lim_{x\to 0}\frac{1-\cos^2 x}{x}$

The Squeeze Theorem

Suppose g(x) < f(x) < h(x) for all x, except maybe x = c. (In other words, f(x) is "squeezed" between g(x) and h(x).)

If $\lim_{x\to c} g(x) = \lim_{x\to c} h(x) = L$, then $\lim_{x\to c} f(x) = L$.

Find $\lim_{x\to 0} x\sin\frac{1}{x}$



Let $g(x)=-|x|$ and $h(x)=|x|$.

If you graph, you can see that $-|x|\le x\sin\frac{1}{x}\le |x|$.

Since $\lim_{x\to 0} g(x)=\lim_{x\to 0} h(x)=0$,

$\lim_{x\to 0} x\sin\frac{1}{x}=0$ as well.